From MIMO to Massive MIMO

Part II : Massive MIMO and Hybrid Transceiver

2024/07/01

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- Massive MIMO and Hybrid Precoder/Combiner
- Narrowband Hybrid Precoder/Combiner Designs
- Wideband Hybrid Precoder/Combiner Designs
- Remarks

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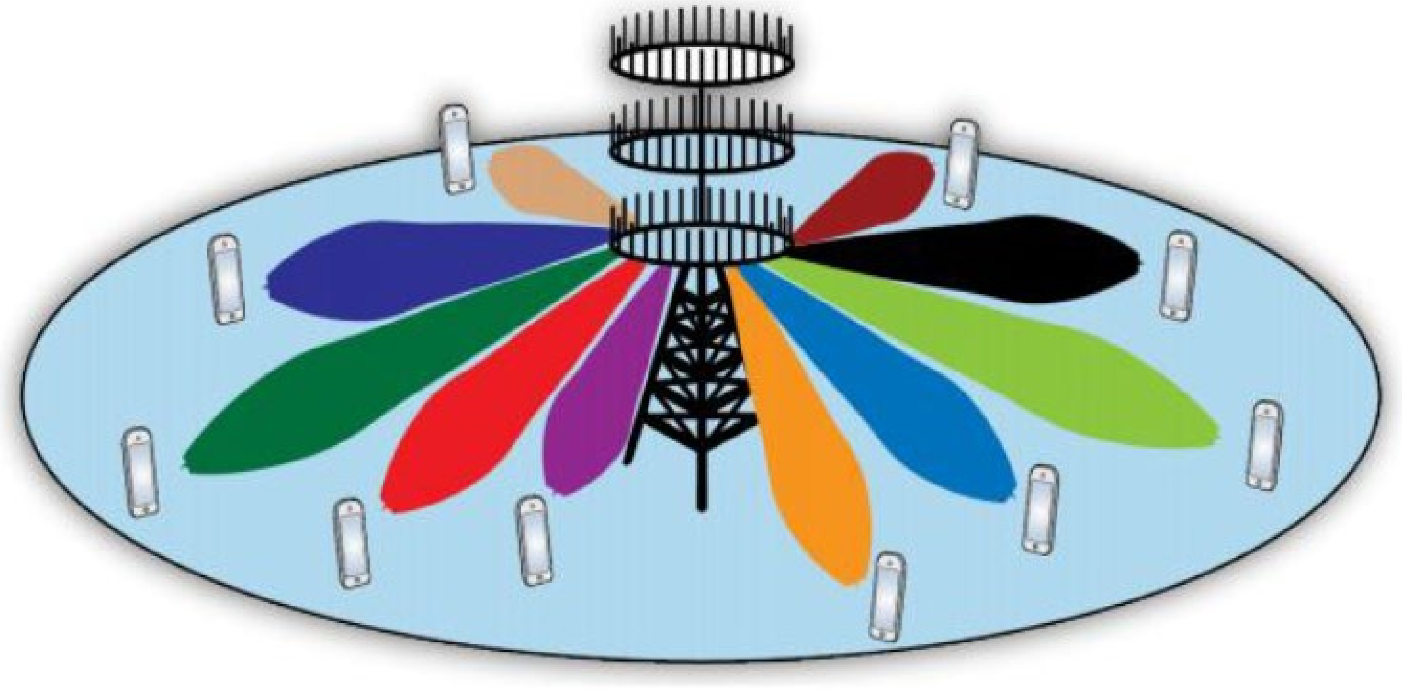
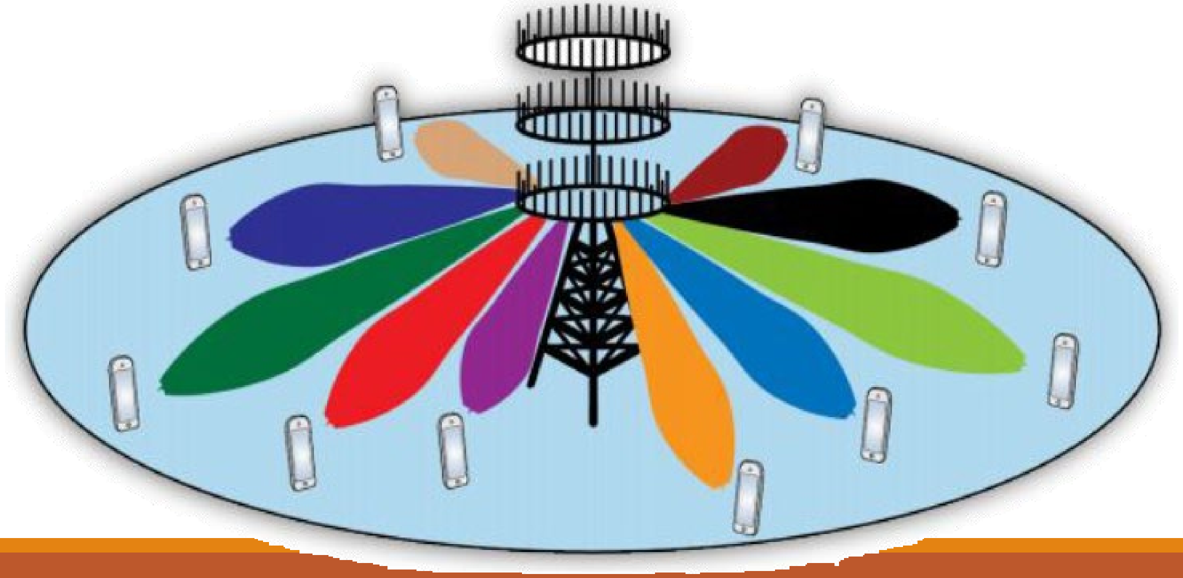
Massive MIMO

- BSs are equipped with a very large number of antennas
- A large number of users are served simultaneously
- Also known as
  - “Large-Scale Antenna Systems”
  - “Very Large MIMO”
  - “Hyper MIMO”
  - “Full-Dimension MIMO”



Challenges of Precoding in Massive MIMO

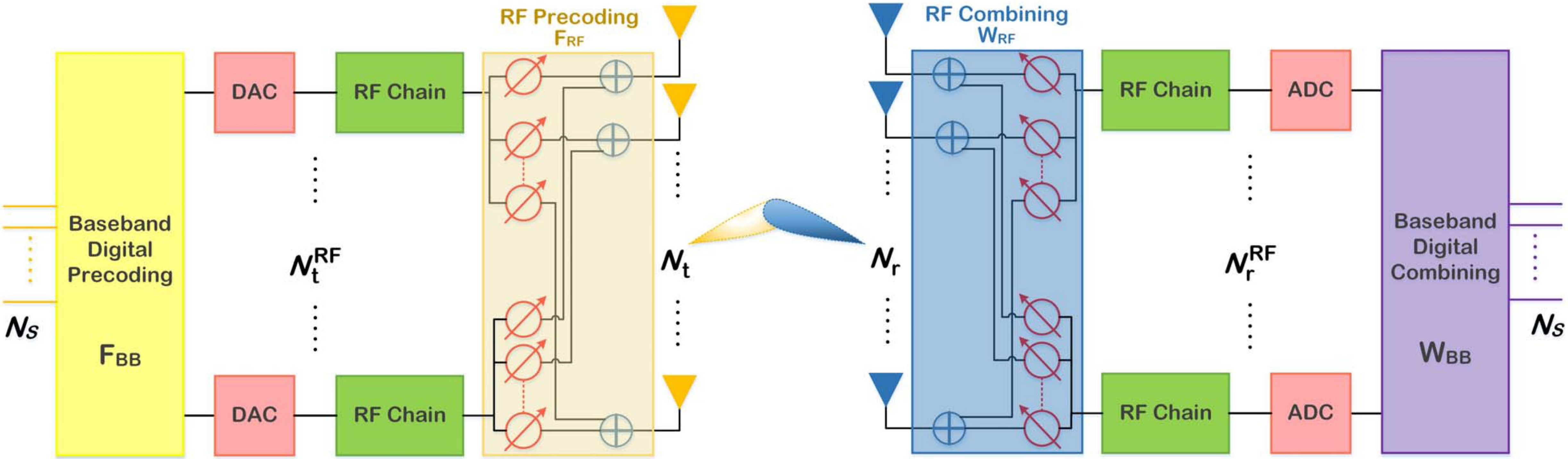
- In Massive MIMO systems with fully digital precoder / combiner
- Number of RF chains, ADCs/DACs and LNAs is identical to Number of antennas
  1. The cost of RF chains and ADCs/DACs are higher especiallyfor mmWave devices
  2. It demands more volume to allocate numerous circuits of RF chain and ADC/DAC
- To tackle the challenges, we may consider
  - Analog beamforming
  - Hybrid Precoder/ Combiner



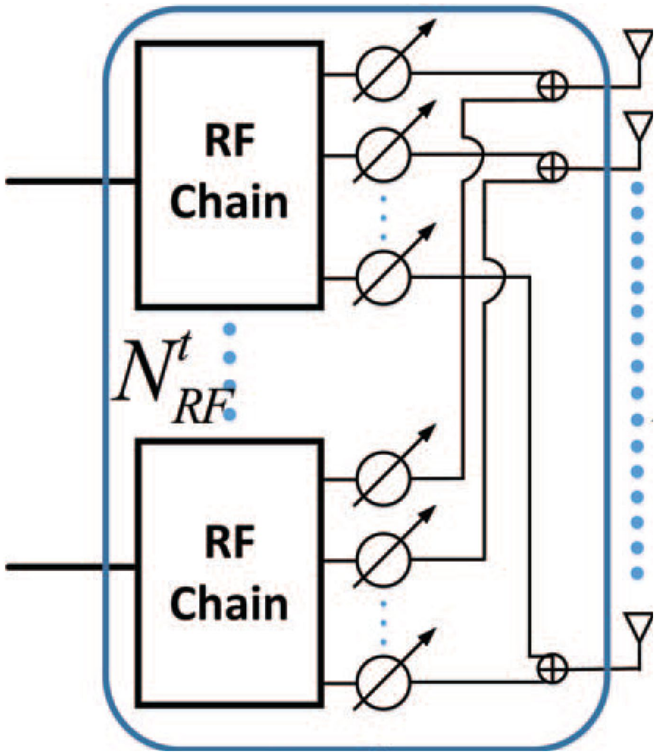
Hybrid Precoder

- ✓ Fully digital precoders requires a large number of radio frequency (RF) chains and analog-to-digital converters (ADCs)
  - Pro: High spectral efficiency
  - Con: High cost, large size, high power consumption
- ✓ Fully analog precoders employing only one RF chain have been applied to mmWave WLAN
  - Pro: low implementation complexity
  - Cons: one spatial stream per cycle
- ✓ Better Tradeoff $\Rightarrow$ Hybrid Precoder !

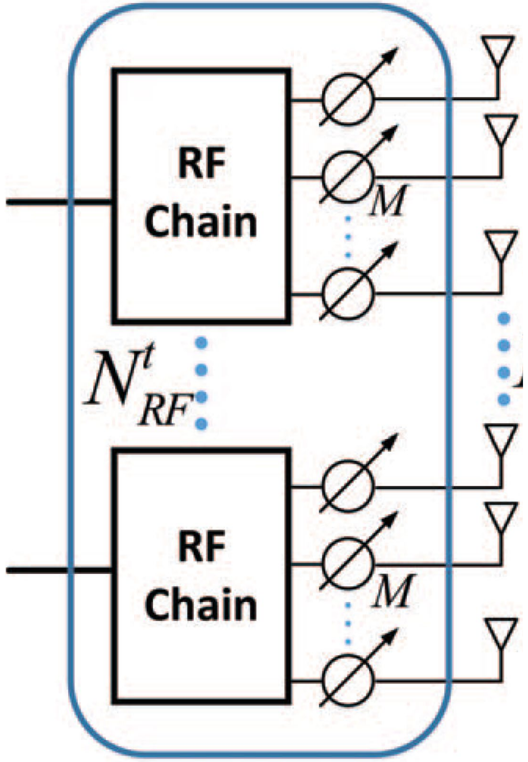
Hybrid Precoder/Combiner



Structures of Hybrid Precoders



Fully Connected Structure



Sub-Connected Structure

- Massive MIMO and Hybrid Precoder/Combiner
- Narrowband Hybrid Precoder/Combiner Designs
- Wideband Hybrid Precoder/Combiner Designs
- Remarks

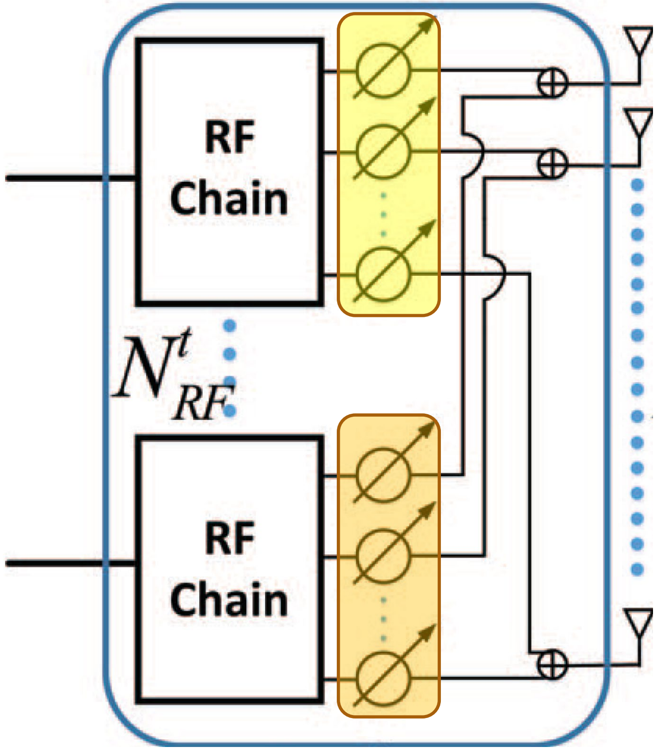
Signal Model of Hybrid Precoder

The transmitted signal vector is

$$\mathbf{x} = \mathbf{F}\mathbf{s} = \mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s}$$

- ✓ $\mathbf{s}$: $N_s \times 1$ vector of data symbols
- ✓ $\mathbf{F}_{BB}$: $N_t^{RF} \times N_s$ digital precoding matrix
- ✓ $\mathbf{F}_{RF}$: $N_t \times N_t^{RF}$ analog precoding matrix
- ✓ Power constraint $||\mathbf{F}_{RF}\mathbf{F}_{BB}||_{\mathrm{F}}^2 = N_s$

Signal Model of Hybrid Precoder

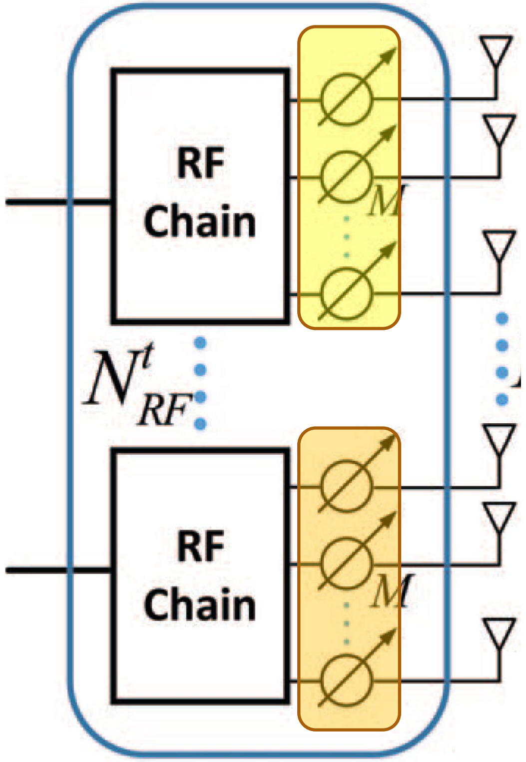


Fully Connected Structure

$$\mathbf{F}_{RF} = \begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix} \quad N_t, \quad N_t^{RF}$$

$$|[\mathbf{F}_{RF}]_{i,j}| = \frac{1}{\sqrt{N_t}}$$

Signal Model of Hybrid Precoder



Sub-Connected Structure

$$\mathbf{F}_{RF} = \begin{bmatrix} \mathbf{f}_1 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ 0 & \mathbf{f}_2 & 0 & 0 \\ 0 & & 0 & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & 0 & \mathbf{f}_{N_t^{RF}} \\ 0 & 0 & 0 & \end{bmatrix}$$

N_t/N_t^{RF}

$$|[\mathbf{f}_k]_{i,j}| = \frac{1}{\sqrt{N_t/N_t^{RF}}}$$

Signal Model of Hybrid Precoder

At the rx, a hybrid receiver is employed and the output is

$$\tilde{\mathbf{y}} = \mathbf{W}_{BB}^{H}\mathbf{W}_{RF}^{H}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{s} + \mathbf{W}_{BB}^{H}\mathbf{W}_{RF}^{H}\mathbf{n}$$

- ✓ $\mathbf{H}$: $N_r \times N_t$ channel matrix
- ✓ $\mathbf{n}$: $N_r \times 1$ AWGN vector
- ✓ $\mathbf{W}_{RF}$: $N_r \times N_r^{RF}$ analog combining matrix
- ✓ $\mathbf{W}_{BB}$: $N_r^{RF} \times N_S$ digital combining matrix

Optimization of Hybrid Precoder

- ✓ Fully-Connected Hybrid Precoder / Combiner
  1. Orthogonal Matching Pursuit (OMP) Algorithm [1]
  2. Manifold Optimization (MO) Based algorithm [2]
- ✓ Sub-Connected Hybrid Precoder / Combiner
  1. Semi-Definite Relaxation (SDR) Based Algorithm [2]
  2. Successive Interference Cancelation (SIC)-Based Algorithm [3]

[1] O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi and R. W. Heath, "Spatially Sparse Precoding in Millimeter Wave MIMO Systems," *IEEE Trans Wireless Communications*, vol. 13, no. 3, pp. 1499-1513, March 2014

[2] X. Yu, et al. "Alternating Minimization Algorithms for Hybrid Precoding in Millimeter Wave MIMO Systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, no. 3, pp. 485-500, April 2016

[3] X. Gao, L. Dai, S. Han, C.-L. I, R. Heath, "Energy-efficient hybrid analog and digital precoding for mmWave MIMO systems with large antenna arrays," *IEEE J. Sel. Areas Commun.*, vol.34, no. 4, pp. 998–1009, April 2016

OMP Algorithm (Fully-Connected)

Achievable rate of the massive MIMO system

$$R = \log\left(\det\left(\mathbf{I} + \frac{E_s}{N_s N_0}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H\right)\right)$$

Problem formulation of precoder optimization:

$$\max\ R = \log\left(\det\left(\mathbf{I} + \frac{E_s}{N_s N_0}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H\right)\right)$$

Subject to $\left|[\mathbf{F}_{RF}]_{i,j}\right| = 1/\sqrt{N_t}$

$$\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_{\mathrm{F}}^2 = N_s$$

OMP Algorithm (Fully-Connected)

Ideally, the optimal solution of the effective precoder $\mathbf{F} = \mathbf{F}_{RF}\mathbf{F}_{BB}$ is

$$\mathbf{F}_{\mathrm{opt}} = \mathbf{V}_{1:N_S}$$

✓ $\mathbf{V}_{1:N_S}$: comprises of first N_s left singular vector of $\mathbf{H}$

Optimization of fully connected hybrid precoder

$$\min \left\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB}\right\|_{\mathrm{F}}$$

$$\text{Subject to } \left|[\mathbf{F}_{RF}]_{i,j}\right| = 1/\sqrt{N_t}$$

$$\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_{\mathrm{F}}^2 = N_s$$

OMP Algorithm (Fully-Connected)

- ✓ In mmWave channel, columns of $\mathbf{F}_{RF}$ can be chosen from $\boldsymbol{a}_t(\theta_0), \boldsymbol{a}_t\,(\theta_1), \dots, \boldsymbol{a}_t(\theta_L)$
- ✓ Nenote a matrix $\mathbf{A}_t = [\boldsymbol{a}_t(\theta_0), \boldsymbol{a}_t\,(\theta_1), \dots, \boldsymbol{a}_t(\theta_L)]$
  - ➢ Construct $\mathbf{F}_{RF}$ by choosing the best N_t^{RF} columns of $\mathbf{A}_t$
- ✓ Optimization problem is now becomes

$$\min \left\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB}\right\|_{\mathrm{F}}$$

$$\text{Subject to } \mathbf{F}_{RF}^{(i)} \in \{\boldsymbol{a}_t(\theta_0), \boldsymbol{a}_t\,(\theta_1), \dots, \boldsymbol{a}_t(\theta_L)\}$$

$$\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_{\mathrm{F}}^2 = N_s$$

  - ➢ Given $\mathbf{F}_{RF}$, $\mathbf{F}_{BB}$cab be found by orthogonal projection of $\mathbf{F}_{\mathrm{opt}}$ on the column space of $\mathbf{F}_{RF}$

Algorithm 1 Spatially Sparse Precoding via Orthogonal Matching Pursuit

Require: $\mathbf{F}_{\text{opt}}$

$\mathbf{F}_{\text{RF}} = \text{Empty Matrix}$

$\mathbf{F}_{\text{res}} = \mathbf{F}_{\text{opt}}$

for $i \leq N_{\text{t}}^{\text{RF}}$ **do**

$\quad \boldsymbol{\Psi} = \mathbf{A}_t^* \mathbf{F}_{\text{res}}$

$\quad k = \arg\max_{\ell=1,\ \dots,\ N_{\text{cl}} N_{\text{ray}}} \left(\boldsymbol{\Psi}\boldsymbol{\Psi}^* \right)_{\ell,\ell}$

$\quad \mathbf{F}_{\text{RF}} = \left[\mathbf{F}_{\text{RF}} \middle| \mathbf{A}_t^{(k)} \right]$

$\quad \boxed{\mathbf{F}_{\text{BB}} = \left(\mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{RF}} \right)^{-1} \mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{opt}}}$ orthogonal projection of $\mathbf{F}_{\text{opt}}$ on the column space of $\mathbf{F}_{RF}$

$\quad \mathbf{F}_{\text{res}} = \frac{\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}}{\| \mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \|_F}$

end for

$\mathbf{F}_{\text{BB}} = \sqrt{N_{\text{s}}} \frac{\mathbf{F}_{\text{BB}}}{\| \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \|_F}$

return $\mathbf{F}_{\text{RF}},\ \mathbf{F}_{\text{BB}}$

OMP Algorithm (Fully-Connected)

- ✓ Given $(\mathbf{F}_{RF}, \mathbf{F}_{BB})$, the hybrid combiner design that minimizes MSE is

$$\min \ \mathbb{E}\left[\left\|\mathbf{s} - \mathbf{W}_{BB}^{H}\mathbf{W}_{RF}^{H}\mathbf{y}\right\|^{2}\right]$$

$$\text{Subject to } \left|[\mathbf{W}_{RF}]_{i,j}\right| = 1/\sqrt{N_r}$$

- ✓ Ideally, the MMSE solution of the effective combiner $\mathbf{W}_{\text{MMSE}}^{H} = \mathbf{W}_{BB}^{H}\mathbf{W}_{\text{RF}}^{H}$ is

$$\mathbf{W}_{\text{MMSE}}^{H} = \mathbb{E}[\mathbf{s}\mathbf{y}^{H}]\mathbb{E}[\mathbf{y}\mathbf{y}^{H}]^{-1}$$

$$= \mathbf{R_s}\mathbf{F}_{BB}^{H}\mathbf{F}_{RF}^{H}\mathbf{H}^{H}\left(\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{R_s}\mathbf{F}_{BB}^{H}\mathbf{F}_{RF}^{H}\mathbf{H}^{H} + \sigma_n^2\mathbf{I}\right)^{-1}$$

OMP Algorithm (Fully-Connected)

- ✓ Mathematically, the optimization in terms of

$$\min \ \mathbb{E}\left[\left\|\mathbf{s}-\mathbf{W}_{BB}^{H}\mathbf{W}_{RF}^{H}\mathbf{y}\right\|^{2}\right]$$

is equivalent to the optimization in terms of

$$\min \ \left\|\mathbb{E}\left[\mathbf{y}\mathbf{y}^{H}\right]^{1/2}\left(\mathbf{W}_{\mathrm{MMSE}}-\mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{BB}}\right)\right\|_{F}$$

- ✓ In mmWave channel, columns of $\mathbf{W}_{RF}$ can be chosen from $\boldsymbol{a}_{\boldsymbol{r}}(\theta_0), \boldsymbol{a}_{\boldsymbol{r}}\ (\theta_1), \dots, \boldsymbol{a}_{\boldsymbol{r}}(\theta_{\boldsymbol{L}})$

- ✓ Nenote a matrix $\mathbf{A}_{\boldsymbol{r}} = [\boldsymbol{a}_{\boldsymbol{r}}(\theta_0), \boldsymbol{a}_{\boldsymbol{r}}\ (\theta_1), \dots, \boldsymbol{a}_{\boldsymbol{r}}(\theta_{\boldsymbol{L}})]$
  - ➢ Construct $\mathbf{W}_{RF}$ by choosing the best N_r^{RF} columns of $\mathbf{A}_{\boldsymbol{r}}$

OMP Algorithm (Fully-Connected)

✓ Optimization problem is now becomes

$$\min \left\| \mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right]^{1/2} \left(\mathbf{W}_{\mathrm{MMSE}} - \mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{BB}}\right) \right\|_F$$

Subject to $\mathbf{W}_{RF}^{(i)} \in \{\boldsymbol{a}_{\boldsymbol{r}}(\theta_0), \boldsymbol{a}_{\boldsymbol{r}}\,(\theta_1), \dots, \boldsymbol{a}_{\boldsymbol{r}}(\theta_L)\}$

1) Given $\mathbf{W}_{MMSE}$, choose a column of $\mathbf{A}_{\boldsymbol{r}}$, such that the inner products of $\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right]^{1/2}\mathbf{W}_{\mathrm{MMSE}}$ and $\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right]^{1/2}\,\mathbf{A}_{\boldsymbol{r}}^{(i)}$ has the largest strength

2) Given $\mathbf{W}_{RF}$, $\mathbf{W}_{BB}$cab be found by orthogonal projection of $\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right]^{1/2}\mathbf{F}_{\mathrm{opt}}$ on the column space of $\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right]^{1/2}\mathbf{F}_{RF}$

Algorithm 2 Spatially Sparse MMSE Combining via Orthogonal Matching Pursuit

Require: $\mathbf{W}_{\mathrm{MMSE}}$

$\mathbf{W}_{\mathrm{RF}} = \text{Empty Matrix}$

$\mathbf{W}_{\mathrm{res}} = \mathbf{W}_{\mathrm{MMSE}}$

for $i \leq N_{\mathrm{r}}^{\mathrm{RF}}$ **do**

$\quad \mathbf{\Psi} = \mathbf{A}_r^* \mathbb{E}\left[\mathbf{y}\mathbf{y}^*\right] \mathbf{W}_{\mathrm{res}}$

$\quad k = \arg\max_{\ell=1,\ \ldots,\ N_{\mathrm{cl}} N_{\mathrm{ray}}} \left(\mathbf{\Psi}\mathbf{\Psi}^*\right)_{\ell,\ell}$

$\quad \mathbf{W}_{\mathrm{RF}} = \left[\mathbf{W}_{\mathrm{RF}} \middle| \mathbf{A}_r^{(k)}\right]$

$\quad \mathbf{W}_{\mathrm{BB}} = \left(\mathbf{W}_{\mathrm{RF}}^* \mathbb{E}\left[\mathbf{y}\mathbf{y}^*\right] \mathbf{W}_{\mathrm{RF}}\right)^{-1} \mathbf{W}_{\mathrm{RF}}^* \mathbb{E}\left[\mathbf{y}\mathbf{y}^*\right] \mathbf{W}_{\mathrm{MMSE}}$

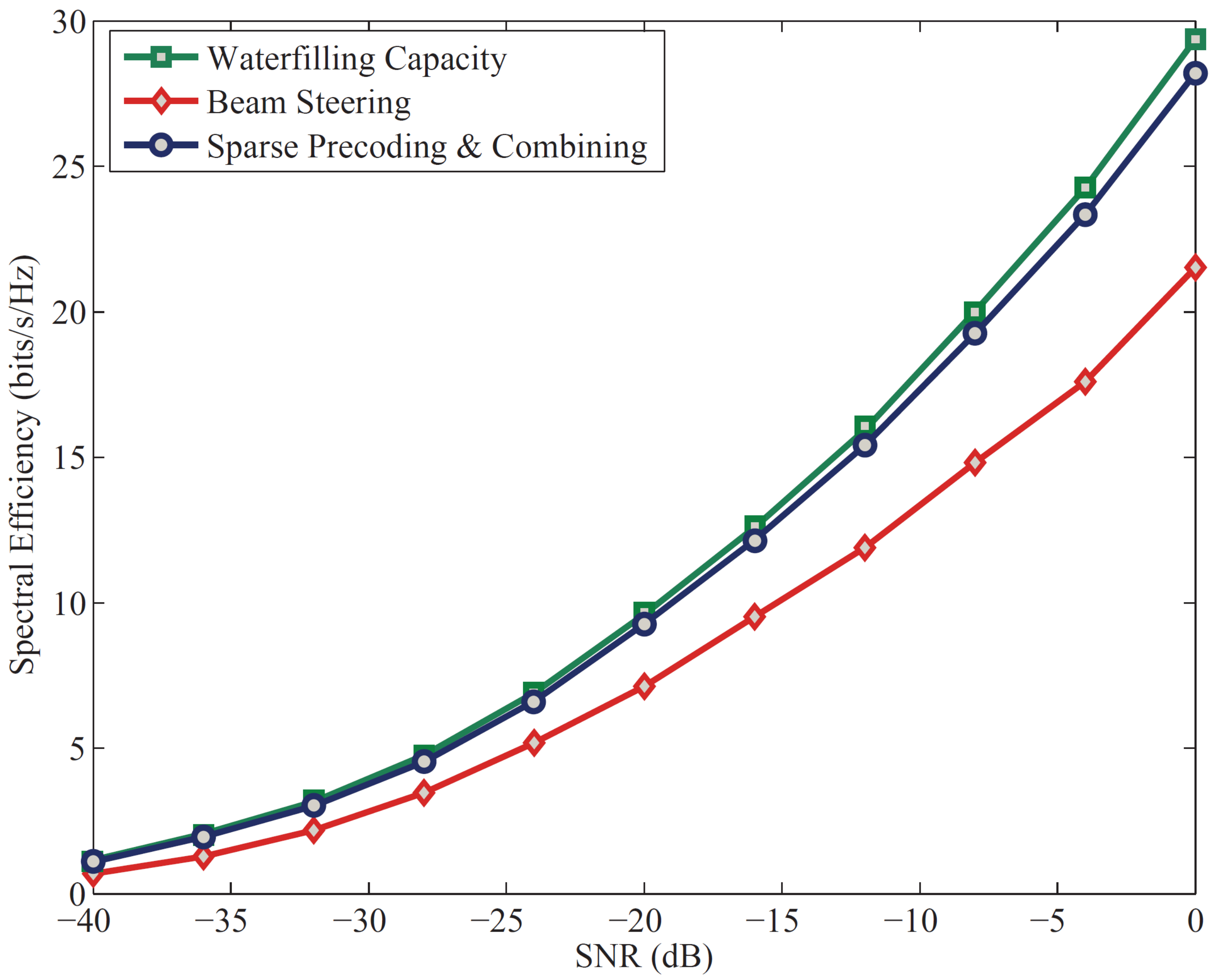
$\quad \mathbf{W}_{\mathrm{res}} = \frac{\mathbf{W}_{\mathrm{MMSE}} - \mathbf{W}_{\mathrm{RF}} \mathbf{W}_{\mathrm{BB}}}{\|\mathbf{W}_{\mathrm{MMSE}} - \mathbf{W}_{\mathrm{RF}} \mathbf{W}_{\mathrm{BB}}\|_F}$

end for

return $\mathbf{W}_{\mathrm{RF}},\ \mathbf{W}_{\mathrm{BB}}$

Simulation Result

- ✓ 256×64 MIMO
- ✓ $N_t^{RF} = N_r^{RF} = 4$



MO Algorithm (Fully-Connected)

Problem formulation for MO algorithm

$$\min \left\| \mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB} \right\|_{\mathrm{F}}$$

$$\text{Subject to } \left| [\mathbf{F}_{RF}]_{i,j} \right| = 1$$

$$\|\mathbf{F}_{RF}\mathbf{F}_{BB}\|_{\mathrm{F}}^{2} = N_s$$

✓ Optimization procedure is similar to the OMP algorithm:

1) Given $\mathbf{F}_{RF}$, $\mathbf{F}_{BB}$cab be found by orthogonal projection of $\mathbf{F}_{\mathrm{opt}}$ on the column space of $\mathbf{F}_{RF}$
2) Design of $\mathbf{F}_{RF}$ employ Manifold Optimization

MO Algorithm (Fully-Connected)

- All feasible solutions of $\mathbf{F}_{RF}$ lie on the space $\mathcal{M}_{cc}^{m}$

$$\mathcal{M}_{cc}^{m} = \{\mathbf{x} \in \mathbb{C}^{m} : |x_1| = |x_2| = \cdots = |x_m| = 1\}$$

- The tangent space at the point $\mathbf{x} \in \mathcal{M}_{cc}^{m}$

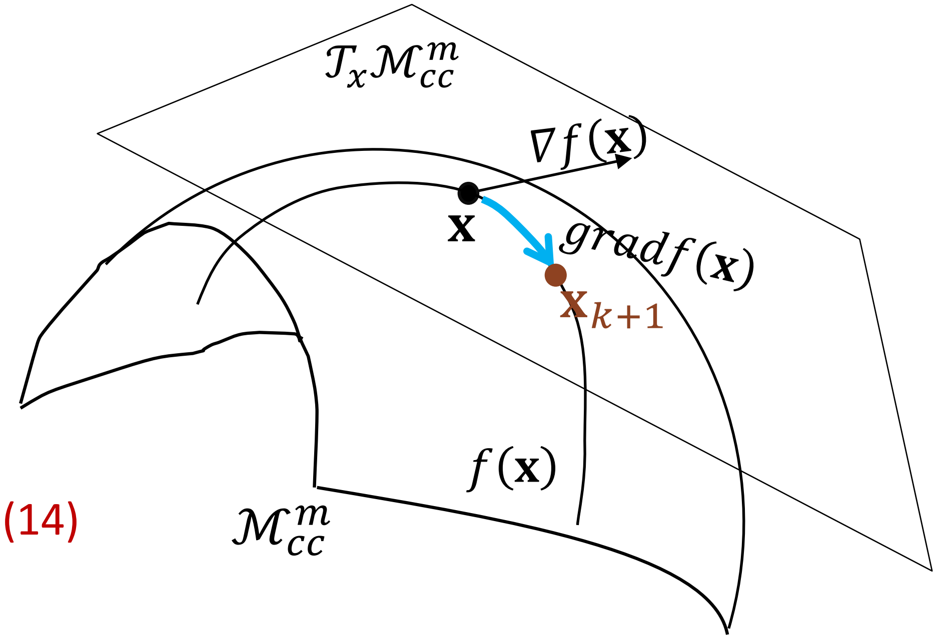
$$T_x\mathcal{M}_{cc}^{m} = \{\mathbf{z} \in \mathbb{C}^{m}: R\{\mathbf{z} \circ \mathbf{x}^*\} = \mathbf{0}_m\}$$

- Euclidean gradient of the cost function

$$\nabla f(\mathbf{x}) = -2(\mathbf{F}_{\mathrm{BB}}^{*} \otimes \mathbf{I}_{N_t})\left[\mathrm{vec}\left(\mathbf{F}_{\mathrm{opt}}\right) - \left(\mathbf{F}_{\mathrm{BB}}^{T} \otimes \mathbf{I}_{\mathrm{N}t}\right)\mathbf{x}\right] \quad (14)$$

- Riemannian gradient at $\mathbf{x} \in \mathbb{C}^{m}$: orthogonal projection of $\nabla f(\mathbf{x})$ onto the tangent space $T_x\mathcal{M}_{cc}^{m}$

$$\mathrm{grad}f(x) = \mathrm{Proj}_{\mathbf{x}}\nabla f(\mathbf{x}) = \nabla f(\mathbf{x}) - \Re\{\nabla f(\mathbf{x}) \circ \mathbf{x}^*\} \circ \mathbf{x} \quad (13)$$

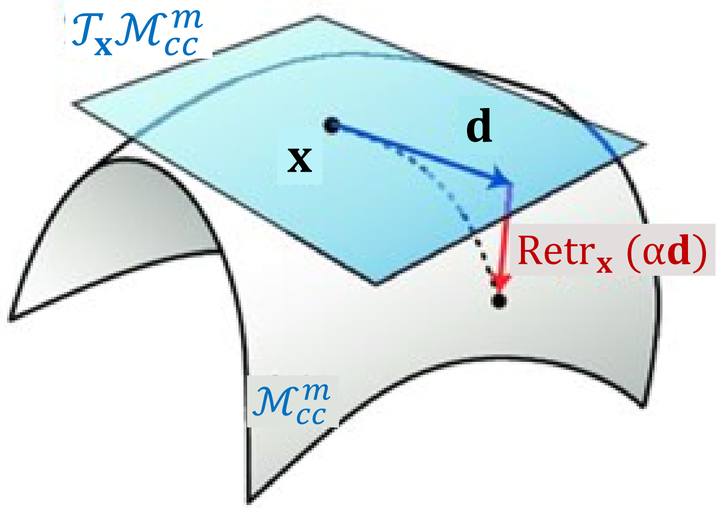


Riemannian gradient

MO Algorithm (Fully-Connected)

- The retraction of a tangent vector $\alpha\mathbf{d}$ at $\mathbf{x} \in \mathcal{M}_{cc}^{m}$

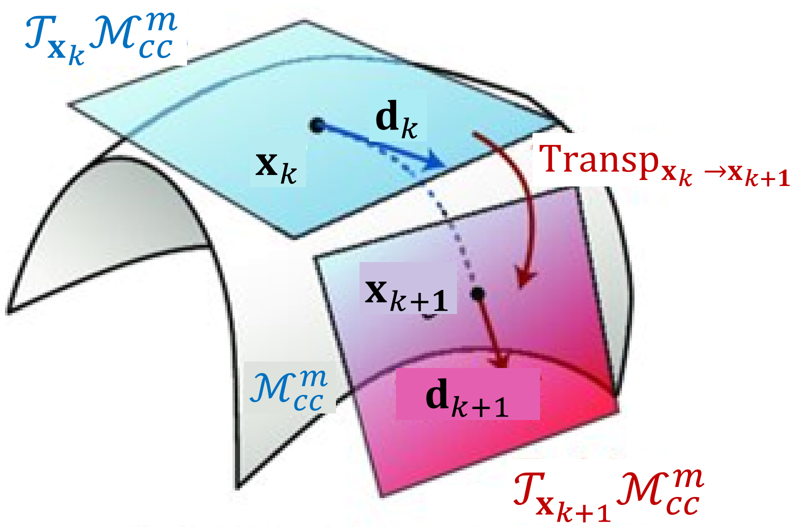
$$\mathrm{Retr}_{\mathbf{x}}(\alpha\mathbf{d}) = \mathrm{vec}\left[\frac{(\mathbf{x}+\alpha\mathbf{d})_i}{|(\mathbf{x}+\alpha\mathbf{d})_i|}\right]. \qquad (15)$$



(a) Retraction

- The transport of a tangent vector $\mathbf{d}$ from $\mathbf{x}_k$ to $\mathbf{x}_{k+1}$

$$\begin{aligned}\mathrm{Transp}_{\mathbf{x}_k \to \mathbf{x}_{k+1}} :& T_{\mathbf{x}_k}\mathcal{M}_{cc}^{m} \to T_{\mathbf{x}_{k+1}}\mathcal{M}_{cc}^{m} : \\ & \mathbf{d} \mapsto \mathbf{d} - \Re\{\mathbf{d} \circ \mathbf{x}_{k+1}^{*}\} \circ \mathbf{x}_{k+1}, \end{aligned} \qquad (16)$$



(b) Vector transport

Algorithm 1 Conjugate Gradient Algorithm for Analog Precoding Based on Manifold Optimization

Input: $\mathbf{F}_{\text{opt}}, \mathbf{F}_{\text{BB}}, \mathbf{x}_0 \in \mathcal{M}_{cc}^m$

$\mathbf{d}_0 = -\text{grad} f(\mathbf{x}_0)$ and $k = 0$;

repeat

Choose Armijo backtracking line search step size α_k;

Find the next point $\mathbf{x}_{k+1}$ using retraction in (15): $\mathbf{x}_{k+1} = \text{Retr}_{\mathbf{x}_k}(\alpha_k \mathbf{d}_k)$;

Determine Riemannian gradient $\mathbf{g}_{k+1} = \text{grad} f(\mathbf{x}_{k+1})$ according to (13) and (14);

Calculate the vector transports $\mathbf{g}_k^+$ and $\mathbf{d}_k^+$ of gradient $\mathbf{g}_k$ and conjugate direction $\mathbf{d}_k$ from $\mathbf{x}_k$ to $\mathbf{x}_{k+1}$;

Choose Polak-Ribiere parameter β_{k+1};

Compute conjugate direction $\mathbf{d}_{k+1} = -\mathbf{g}_{k+1} + \beta_{k+1}\mathbf{d}_k^+$;

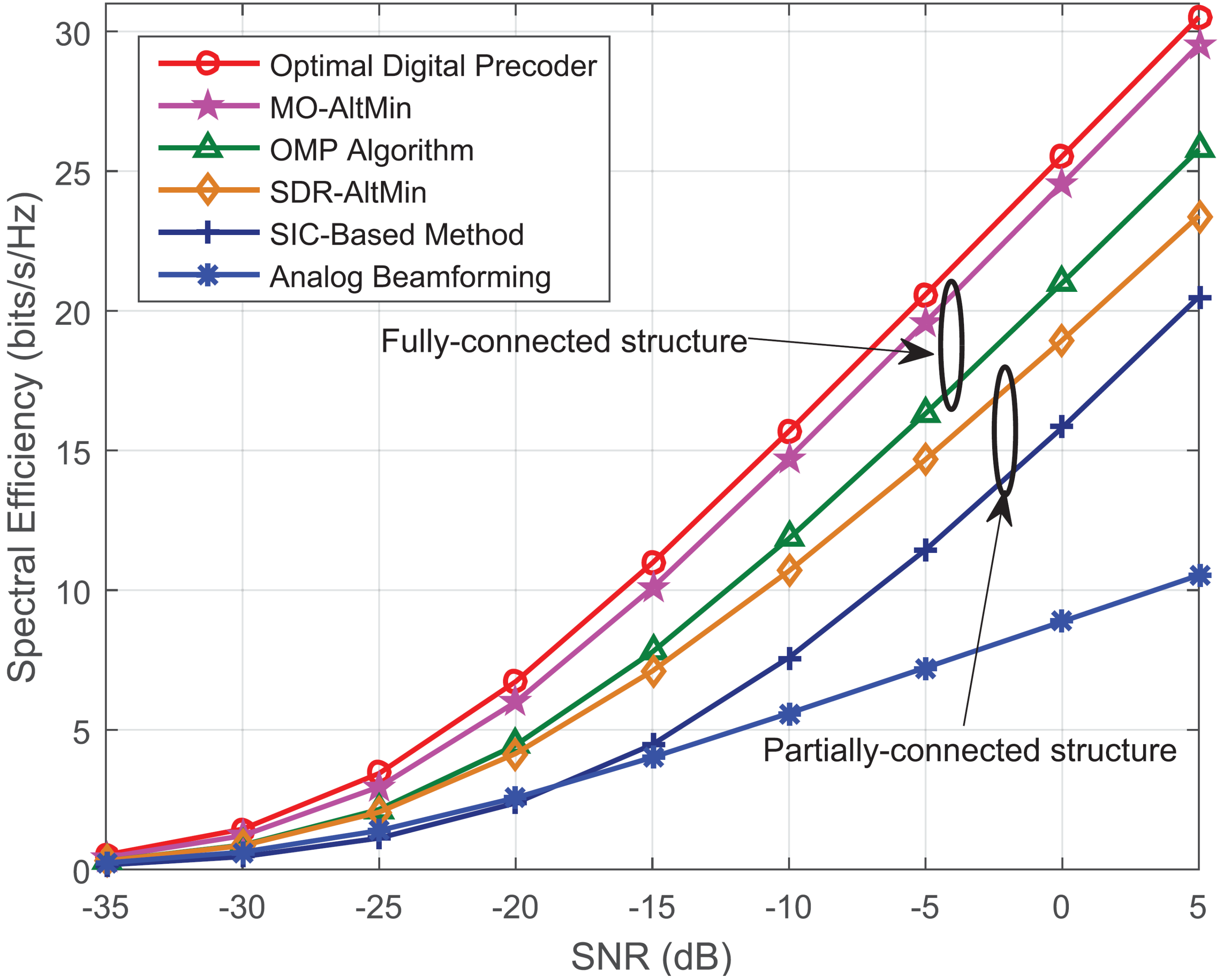
$k \leftarrow k + 1$;

until a stopping criterion triggers.

Vector transport

Performance Comparison

- ✓ 144×36 MIMO
- ✓ $N_t^{RF} = N_r^{RF} = 3$



SIC-Based Method (Sub-Connected)

Consider a massive MIMO system

- ✓ $N_t = N_t^{RF} \cdot M$
- ✓ $N_S = N_t^{RF}$ data streams are transmitted
- ✓ Digital precoder is assumed digital : $\mathbf{F}_{BB} = diag(d_1, d_2, \dots, d_{N_S})$
- ✓ Analog precoder :

$$\mathbf{F}_{RF} = \begin{bmatrix} \mathbf{f}_1 & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ 0 & \mathbf{f}_2 & 0 & 0 \\ 0 & & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & & 0 \\ 0 & 0 & 0 & \mathbf{f}_{N_S} \\ 0 & 0 & 0 & \end{bmatrix} \quad M$$

$$|[\mathbf{f}_k]_{i,j}| = \frac{1}{\sqrt{N_t / N_t^{RF}}}$$

SIC-Based Method (Sub-Connected)

The composite of precoding matrix becomes

$$\mathbf{P} \triangleq \mathbf{F}_{RF}\mathbf{F}_{BB} = \begin{bmatrix} \mathbf{f}_1 & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & \mathbf{f}_2 & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \ddots & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \mathbf{f}_{N_S} \end{bmatrix} \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_{N_S} \end{bmatrix} \triangleq \begin{bmatrix} \widetilde{\mathbf{p}}_1 & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & \widetilde{\mathbf{p}}_2 & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \ddots & \begin{matrix}0\\0\end{matrix} \\ \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \begin{matrix}0\\0\end{matrix} & \widetilde{\mathbf{p}}_{N_S} \end{bmatrix}, \qquad \widetilde{\mathbf{p}}_i = d_i \mathbf{f}_i$$

$$\mathbf{p}_1 \quad \mathbf{p}_2 \quad \cdots \quad \mathbf{p}_{N_S}$$

SIC-Based Method (Sub-Connected)

Define $\mathbf{P} \triangleq \mathbf{F}_{RF}\mathbf{F}_{BB} \triangleq [\mathbf{P}_{N_s-1} \quad \mathbf{p}_{N_s}]$

Achievable rate of the hybrid precoding system:

$$R = \log\left(\det\left(\mathbf{I} + \frac{E_s}{N_s N_0}\mathbf{H}\mathbf{F}_{RF}\mathbf{F}_{BB}\mathbf{F}_{BB}^H\mathbf{F}_{RF}^H\mathbf{H}^H\right)\right)$$

$$= \log\left(\det\left(\mathbf{T}_{N_s-1}\right)\right) + \log\left(\det\left(\mathbf{I} + \frac{E_s}{N_s N_0}\mathbf{T}_{N_s-1}^{-1}\mathbf{H}\mathbf{p}_{N_s}\mathbf{p}_{N_s}^H\mathbf{H}^H\right)\right)$$

$$= \log\left(\det\left(\mathbf{T}_{N_s-1}\right)\right) + \log\left(\det\left(\mathbf{I} + \frac{E_s}{N_s N_0}\mathbf{p}_{N_s}^H\mathbf{H}^H\mathbf{T}_{N_s-1}^{-1}\mathbf{H}\mathbf{p}_{N_s}\right)\right)$$

- ✓ $\mathbf{T}_{N_s-1} = \mathbf{I} + \frac{E_s}{N_s N_0}\mathbf{H}\mathbf{P}_{N_s-1}\mathbf{P}_{N_s-1}^H\mathbf{H}^H$
- ✓ $\det(\mathbf{I} + \mathbf{X}\mathbf{Y}) = \det(\mathbf{I} + \mathbf{Y}\mathbf{X})$

SIC-Based Method (Sub-Connected)

After N_s decomposition, achievable rate becomes

$$R = \sum_{n=1}^{N_s} \log\left(1 + \frac{E_s}{N_s N_0}\mathbf{p}_n^H \mathbf{H}^H \mathbf{T}_{n-1}^{-1} \mathbf{H}\mathbf{p}_n\right)$$

- ✓ $\mathbf{T}_n = \mathbf{I} + \frac{E_s}{N_0}\mathbf{H}\mathbf{P}_n\mathbf{P}_n^H\mathbf{H}^H$
- ✓ $\mathbf{T}_0 = \mathbf{I}_{N_s}$
- ✓ $\mathbf{P}_n = [\mathbf{P}]_{:,1:\boldsymbol{n}}$
- ✓ Each column $\mathbf{p}_n$ can be optimized individually

SIC-Based Method (Sub-Connected)

Optimization of $\mathbf{p}_n$ (or $\widetilde{\mathbf{p}}_n$):

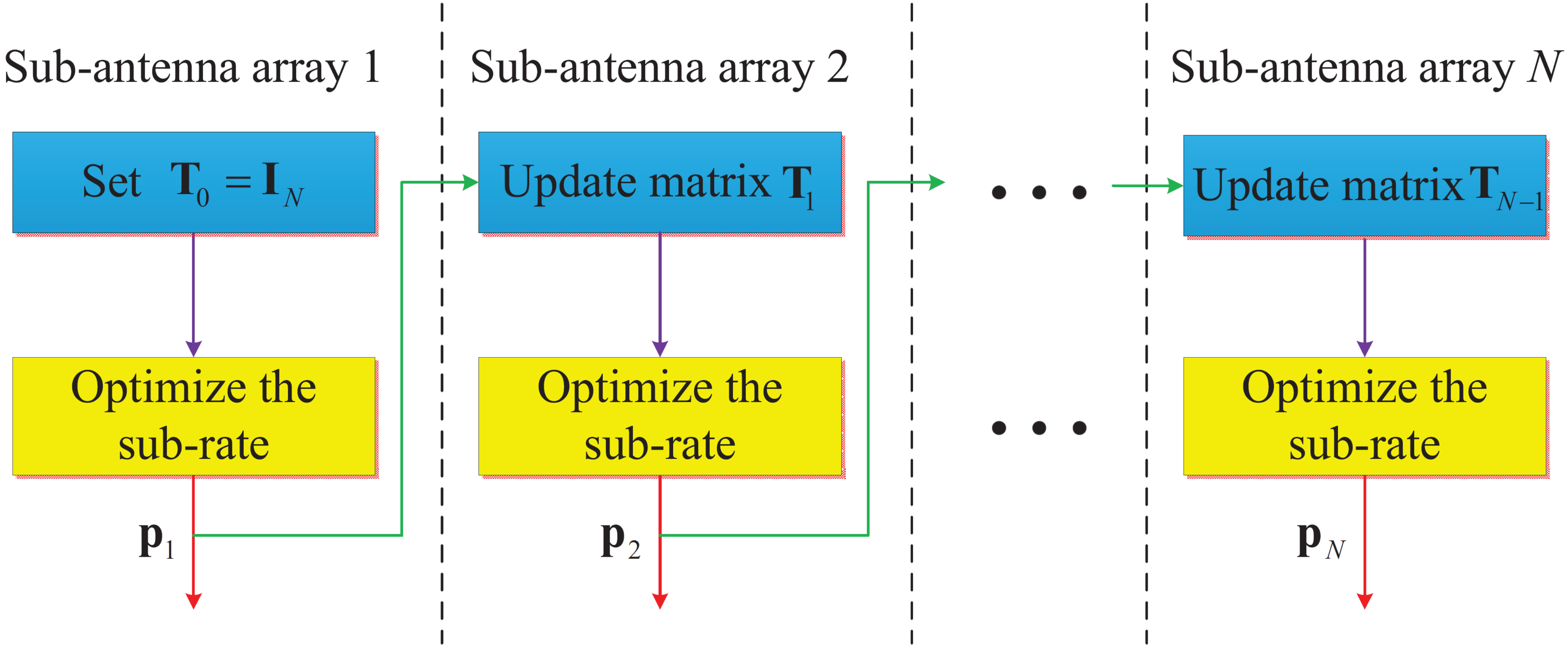
$$\max \quad \log\left(1+\frac{E_s}{N_s N_0}\mathbf{p}_n^H\mathbf{H}^H\mathbf{T}_{n-1}^{-1}\mathbf{H}\mathbf{p}_n\right)=\log\left(1+\frac{E_s}{N_s N_0}\widetilde{\mathbf{p}}_n^H\widetilde{\mathbf{G}}_{n-1}\widetilde{\mathbf{p}}_n\right)$$

$$\text{Subject to } \left|[\widetilde{\mathbf{p}}_n]_1\right|=\left|[\widetilde{\mathbf{p}}_n]_2\right|=\cdots=\left|[\widetilde{\mathbf{p}}_n]_M\right|$$

$$\sum_{n=1}^{N_s}\sum_{m=1}^{M}\left|[\widetilde{\mathbf{p}}_n]_m\right|^2\le N_s$$

- ✓ $\tilde{\mathbf{G}}_{n-1}=\mathbf{R}_n\mathbf{H}^H\mathbf{T}_{n-1}^{-1}\mathbf{H}\mathbf{R}_n^H$
- ✓ $\mathbf{R}_n=\left[\mathbf{0}_{M\times M(n-1)}\quad \mathbf{I}_{M\times M}\quad \mathbf{0}_{M\times M(N-n)}\right]$

SIC-Based Method (Sub-Connected)



SIC-Based Method (Sub-Connected)

- Let $\mathbf{v}_n$ the first column of singular matrix of $\widetilde{\mathbf{G}}_{n-1}$
- It is proven that the optimal precoder is

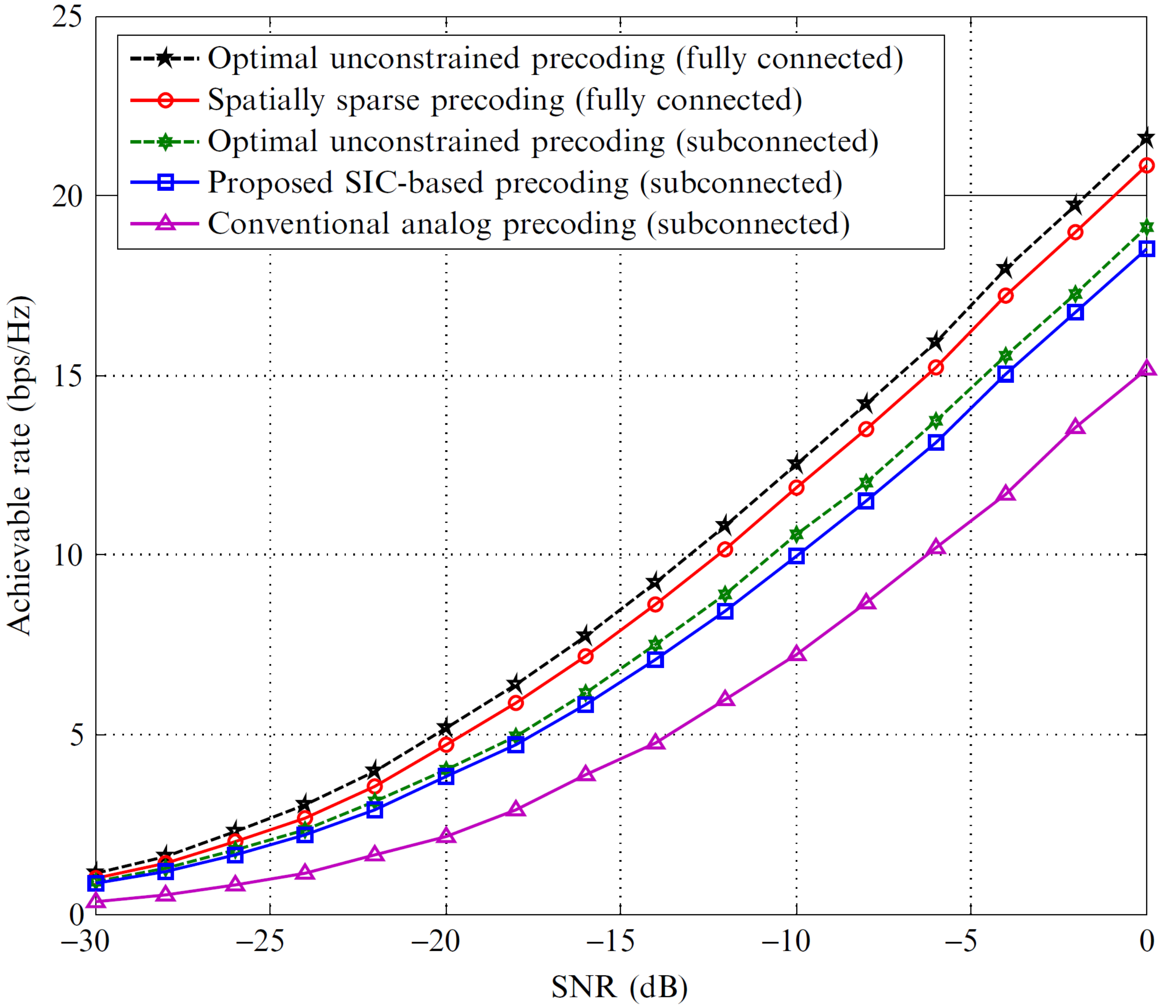
$$\mathbf{f}_n = \frac{1}{\sqrt{M}} e^{j\measuredangle \mathbf{v}_n}$$

$$d_n = \frac{\|\mathbf{v}_n\|_1}{\sqrt{M}}$$

$$\Longleftrightarrow \qquad \widetilde{\mathbf{p}}_n = \frac{\|\mathbf{v}_n\|_1}{M} e^{j\measuredangle \mathbf{v}_n}$$

Performance Comparison

- ✓ 64×16 MIMO
- ✓ $N_t^{RF} = 8$



SDR-Based Method (Sub-Connected)

Consider a massive MIMO system

- ✓ Analog precoder :

$$\mathbf{F}_{RF} = \begin{bmatrix} \mathbf{f}_1 & 0 & 0 & 0 \\ 0 & \mathbf{f}_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{f}_{N_S} \end{bmatrix}$$

N_t/N_t^{RF}

$$|[\mathbf{f}_k]_{i,j}| = 1$$

$$\mathbf{F}_{RF}^{\mathrm{H}}\mathbf{F}_{RF} = \frac{N_t}{N_t^{RF}}\mathbf{I}_{N_S}$$

- ✓ Power constraint :

$$\|\mathbf{F}_{RF}\mathbf{F}_{\mathrm{BB}}\|_F^2 = \frac{N_t}{N_t^{RF}}\|\mathbf{F}_{\mathrm{BB}}\|_F^2 = N_S \iff \|\mathbf{F}_{\mathrm{BB}}\|_F^2 = \frac{N_t^{RF}N_S}{N_t}$$

SDR-Based Method (Sub-Connected)

Problem Formulation:

$$\min \left\| \mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB} \right\|_{\mathrm{F}}$$

$$\text{Subject to } \mathbf{F}_{RF} \in \mathcal{A}_p$$

$$\left\| \mathbf{F}_{\mathrm{BB}} \right\|_F^2 = \frac{N_t^{RF} N_S}{N_t}$$

- ✓ $\mathcal{A}_p$: set of all feasible solutions of sub-connected analog precoder
- ✓ Alternative optimizations of $\mathbf{F}_{RF}$ and $\mathbf{F}_{BB}$

SDR-Based Method (Sub-Connected)

Step 1: Optimization of $\mathbf{F}_{RF}$ given $\mathbf{F}_{BB}$:

$$\min \left\|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB}\right\|_{\mathrm{F}}$$

$$\text{Subject to } \mathbf{F}_{RF} \in \mathcal{A}_p$$

$$\mathbf{F}_{RF} = \begin{bmatrix} e^{j\theta_1} & 0 & 0 & 0 \\ \vdots & & & \\ e^{j\theta_M} & 0 & 0 & 0 \\ 0 & e^{j\theta_{M+1}} & 0 & 0 \\ 0 & \vdots & 0 & 0 \\ 0 & e^{j\theta_{2M}} & \ddots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & e^{j\theta_{N_t}} \end{bmatrix} \quad \overset{M = \frac{N_t}{N_t^{RF}}}{\Longleftrightarrow} \quad \min_{\theta_i} \left\| \left(\mathbf{F}_{\mathrm{opt}}\right)_{i,:} - e^{j\theta_i}\left(\mathbf{F}_{BB}\right)_{l,:} \right\|_{\mathrm{F}}$$

$$\text{where } l = \left\lfloor i\frac{N_t^{RF}}{N_t} \right\rfloor = \left\lfloor \frac{i}{M} \right\rfloor$$

$$\text{for } i = 1,2,\ldots,N_t$$

SDR-Based Method (Sub-Connected)

Step 1: Optimization of $\mathbf{F}_{RF}$ given $\mathbf{F}_{BB}$:

$$\min_{\theta_i} \left\| \left(\mathbf{F}_{\text{opt}}\right)_{i,:} - e^{-j\theta_i}(\mathbf{F}_{BB})_{l,:} \right\|_{\text{F}} \quad \underset{\Longleftrightarrow}{\text{elementwise optmization}} \quad \theta_i = \measuredangle(\mathbf{F}_{BB})_{i,l} = \measuredangle\left\{\left(\mathbf{F}_{\text{opt}}\right)_{i,:}(\mathbf{F}_{BB})^H_{l,:}\right\}$$

where $l = \left\lfloor i\frac{N_t^{RF}}{N_t} \right\rfloor = \left\lfloor \frac{i}{M} \right\rfloor$

for $i = 1,2,\dots,N_t$

for $i = 1,2,\dots,N_t$

$l = \left\lfloor i\frac{N_t^{RF}}{N_t} \right\rfloor = \left\lfloor \frac{i}{M} \right\rfloor$

SDR-Based Method (Sub-Connected)

Step 2: Optimization of $\mathbf{F}_{BB}$ given $\mathbf{F}_{RF}$:

$$\min \left\| \mathbf{F}_{\text{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB} \right\|_F^2$$

$$\text{Subject to } \|\mathbf{F}_{\text{BB}}\|_F^2 = \frac{N_t^{RF} N_S}{N_t}$$

✓
$$\begin{aligned}\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 &= \|\text{vec}(\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}})\|_2^2 \\ &= \|\text{vec}(\mathbf{F}_{\text{opt}}) - \text{vec}(\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}})\|_2^2 \\ &= \|\text{vec}(\mathbf{F}_{\text{opt}}) - (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\text{RF}})\text{vec}(\mathbf{F}_{\text{BB}})\|_2^2 .\end{aligned}$$

✓ Define
$$\mathbf{f} = \text{vec}(\mathbf{F}_{\text{opt}})$$
$$\mathbf{b} = \text{vec}(\mathbf{F}_{\text{BB}})$$
$$\mathbf{E} = \mathbf{I}_{N_s} \otimes \mathbf{F}_{\text{RF}}$$

$$\Longleftrightarrow \quad \left\| \mathbf{F}_{\text{opt}} - \mathbf{F}_{RF}\mathbf{F}_{BB} \right\|_F^2 = \|\mathbf{f} - \mathbf{E}\mathbf{b}\|_F^2$$

SDR-Based Method (Sub-Connected)

Step 2: Optimization of $\mathbf{F}_{BB}$ given $\mathbf{F}_{RF}$:

$$\min \|t\mathbf{f} - \mathbf{E}\mathbf{b}\|_F^2$$

$$\text{Subject to } \|\mathbf{f}\|^2 = \frac{N_t^{RF} N_S}{N_t}$$

$$t^2 = 1$$

← t is an auxiliary variable

QCQP Problem
Non-convax

✓ $\|t\mathbf{f} - \mathbf{E}\mathbf{b}\|_2^2 = \begin{bmatrix} \mathbf{b}^H & t \end{bmatrix} \begin{bmatrix} \mathbf{E}^H\mathbf{E} & -\mathbf{E}^H\mathbf{f} \\ -\mathbf{f}^H\mathbf{E} & \mathbf{f}^H\mathbf{f} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ t \end{bmatrix}.$

✓ $\|\mathbf{b}\|_2^2 = \begin{bmatrix} \mathbf{b}^H & t \end{bmatrix} \begin{bmatrix} \mathbf{I}_{N_{\mathrm{RF}}^t N_s} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ t \end{bmatrix}$

✓ $t^2 = \begin{bmatrix} \mathbf{b}^H & t \end{bmatrix} \begin{bmatrix} \mathbf{0}_{N_{\mathrm{RF}}^t N_s} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ t \end{bmatrix}$

SDR-Based Method (Sub-Connected)

Step 2: Optimization of $\mathbf{F}_{BB}$ given $\mathbf{F}_{RF}$:

$$\min \operatorname{tr}(\mathbf{CY})$$

$$\text{Subject to } \operatorname{tr}(\mathbf{A}_1\mathbf{Y}) = \frac{N_t^{RF} N_S}{N_t}$$

$$\operatorname{tr}(\mathbf{A}_2\mathbf{Y}) = 1$$

$$\mathbf{Y} \succcurlyeq 0, \operatorname{rank}(\mathbf{Y}) = 1$$

By relaxing the constraint:
rank(**Y**)=1
It becomes a **semidefinite relaxation (SDR)** problem, and can be solved using CVX

- ✓ $\mathbf{y} = \begin{bmatrix} \mathbf{b} \\ t \end{bmatrix}, \mathbf{Y} = \mathbf{y}\mathbf{y}^H$
- ✓ $\mathbf{A}_1 = \begin{bmatrix} \mathbf{I}_{n-1} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} \mathbf{0}_{n-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix},$
- ✓ $\mathbf{C} = \begin{bmatrix} (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}})^H (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}}) & -(\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}})^H \mathbf{f} \\ -\mathbf{f}^H (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\mathrm{RF}}) & \mathbf{f}^H \mathbf{f} \end{bmatrix}$

SDR-Based Method (Sub-Connected)

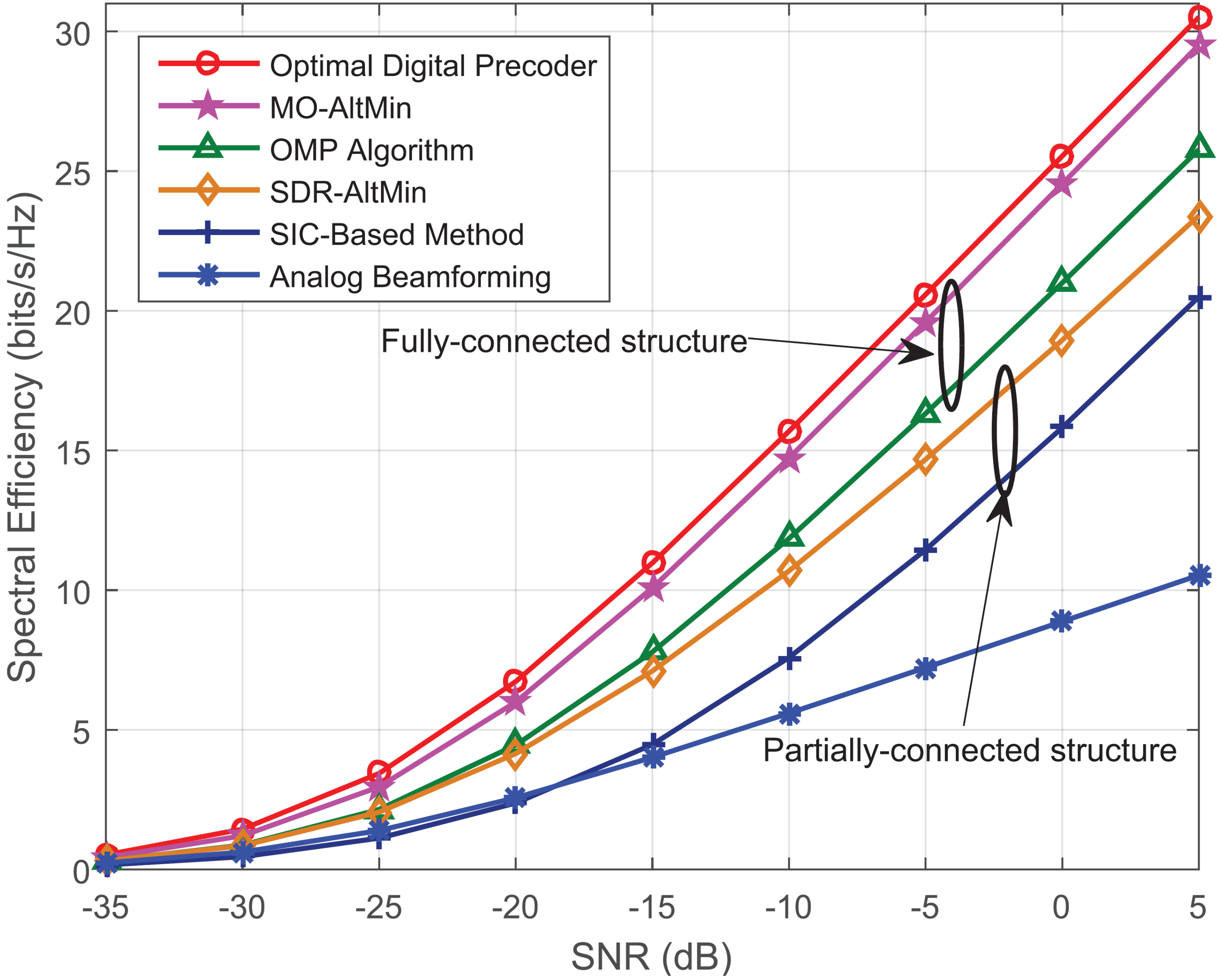
SDR-AltMin Algorithm: Semidefinite Relaxation Based Hybrid Precoding for the Partially-connected Structure

Input: $\mathbf{F}_{\text{opt}}$

1: Construct $\mathbf{F}_{\text{RF}}^{(0)}$ with random phases and set $k = 0$;
2: **repeat**
3: Fix $\mathbf{F}_{\text{RF}}^{(k)}$, solving $\mathbf{F}_{\text{BB}}^{(k)}$ using SDR (36);
4: Fix $\mathbf{F}_{\text{BB}}^{(k)}$, and update $\mathbf{F}_{\text{RF}}^{(k+1)}$ by (33);
5: $k \leftarrow k + 1$;
6: **until** a stopping criterion triggers.

Performance Comparison

- ✓ 144×36 MIMO
- ✓ $N_t^{RF} = N_r^{RF} = 3$



- Massive MIMO and Hybrid Precoder/Combiner
- Narrowband Hybrid Precoder/Combiner Designs
- **Wideband Hybrid Precoder/Combiner Designs**
- Remarks

Wideband Hybrid Precoding System

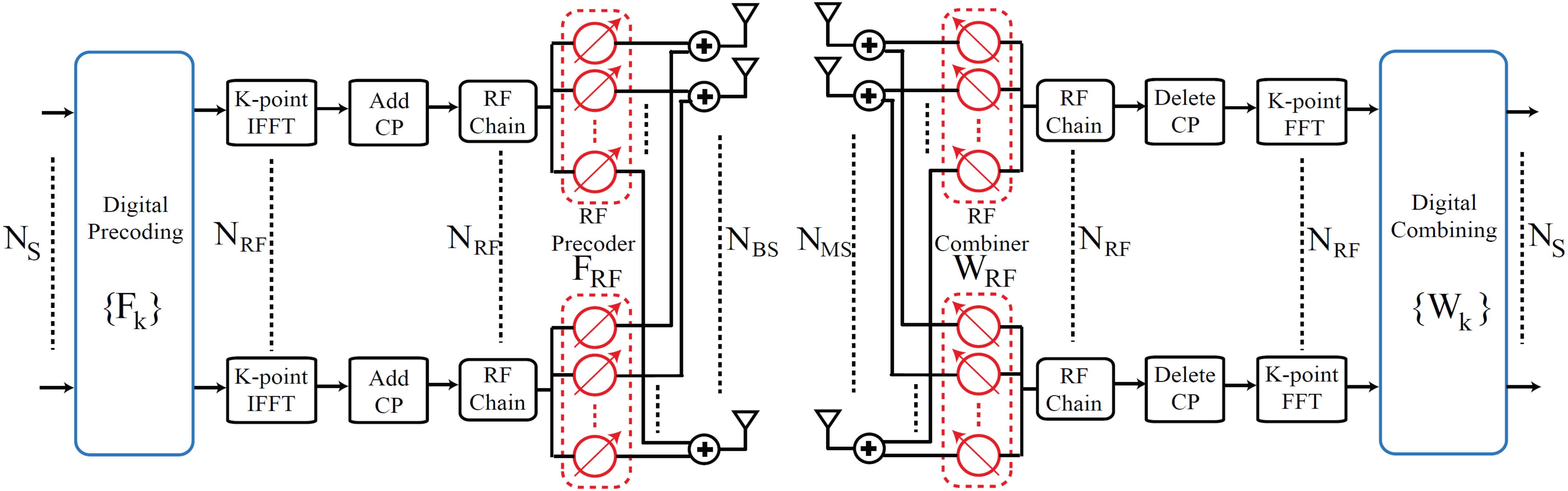
Consider an OFDM-MIMO system with hybrid precoder/combiner:

- ✓ Tx (BS) has N_{BS} antennas and $N_{\mathbf{RF}}$ RF chains
- ✓ Rx (MS) has $N_{\mathbf{M}S}$ antennas and $N_{\mathbf{RF}}$ RF chains
- ✓ N_s length-K data symbols are transmitted simultaneously
- ✓ The OFDM system has K sub-carriers
- ✓ The composite precoding matrix $\mathbf{F}[k] = \mathbf{F}_{RF}\mathbf{F}_{BB}[k]$ is semi-unitary

$$\mathbf{F}[k] \in \mathcal{U}_{N_{BS}\times N_S} = \{\mathbf{U} \in \mathbb{C}^{N_{BS}\times N_S} | \mathbf{U}\mathbf{U}^H = \mathbf{I}\}$$

A. Alkhateeb and R. W. Heath, "Gram Schmidt based greedy hybrid precoding for frequency selective millimeter wave MIMO systems", *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP) Shanghai China*, pp. 3396-3400, Mar. 2016.

Wideband Hybrid Precoding System



Wideband Hybrid Precoding System

Signal received at subcarrier k:

$$\mathbf{y}[k] = \mathbf{W}[k]^*\mathbf{W}_{\mathrm{RF}}^*\mathbf{H}[k]\mathbf{F}_{\mathrm{RF}}\mathbf{F}[k]s[k] + \mathbf{W}[k]^*\mathbf{W}_{\mathrm{RF}}^*\mathbf{n}[k]$$

Achievable rate of the MIMO-OFDM system

$$R = \sum_{k=1}^{K} \log\left(\det\left(\mathbf{I} + \frac{E_s}{KN_sN_0}\mathbf{H}[k]\mathbf{F}_{RF}\mathbf{F}_{BB}[k]\mathbf{F}_{BB}^H[k]\mathbf{F}_{RF}^H\mathbf{H}[k]^H\right)\right)$$

The precoder design aims to maximize R subject to constraints

$$\left|[\mathbf{F}_{RF}]_{i,j}\right| = 1$$

$$\mathbf{F}_{RF}\mathbf{F}_{BB}[k] \in \mathcal{U}_{N_{BS}\times N_S}$$

Wideband Hybrid Precoding System

Take SVD of $\mathbf{H}[k]$:

$$\mathbf{H}[k] = \mathbf{U}[k]\boldsymbol{\Sigma}[k]\mathbf{V}[k]^H$$

Given $\mathbf{F}_{RF}$, the optimal digital precoder is

$$\mathbf{F}_{BB}[k] = \left(\mathbf{F}_{RF}^H\mathbf{F}_{RF}\right)^{-\frac{1}{2}}\left[\overline{\mathbf{V}}[k]\right]_{:,1:N_S}, \qquad k = 1,2,\dots,K$$

✓ $\overline{\mathbf{V}}[k]$ comes from the SVD of

$$\boldsymbol{\Sigma}[k]\mathbf{V}[k]^H\mathbf{F}_{RF}\left(\mathbf{F}_{RF}^H\mathbf{F}_{RF}\right)^{-\frac{1}{2}} = \overline{\mathbf{U}}[k]\,\overline{\boldsymbol{\Sigma}}[k]\overline{\mathbf{V}}[k]^H$$

Wideband Hybrid Precoding System

Three algorithms were proposed to design $\mathbf{F}_{RF}$:

- ✓ Direct greedy hybrid precoding
- ✓ Gram-Schmidt greedy hybrid precoding
- ✓ Approximate Gram-Schmidt based greedy hybrid precoding

Wideband Hybrid Precoding System

(1) Direct greedy hybrid precoding

- ✓ The columns of $\mathbf{F}_{RF}$ are optimized successively
- ✓ In the i-th iteration, the i-th column of $\mathbf{F}_{RF}$ are optimized in terms of

$$\max_{\mathbf{f}_n \in \mathcal{F}_{RF}} R^{(i)}(\mathbf{f}_n) = \sum_{k=1}^{K} \sum_{l=1}^{i} \log\left(1 + \frac{E_s}{K N_s N_0} \lambda_l \left(\mathbf{H}[k] \hat{\mathbf{F}}_{RF}^{(i,n)} \left(\hat{\mathbf{F}}_{RF}^{(i,n)H} \hat{\mathbf{F}}_{RF}^{(i,\boldsymbol{n})}\right)^{-1} \hat{\mathbf{F}}_{RF}^{(i,n)H} \mathbf{H}[k]^H\right)\right)$$

- ✓ $\hat{\mathbf{F}}_{RF}^{(i,n)} = \left[\hat{\mathbf{F}}_{RF}^{(i-1)} \quad \mathbf{f}_n\right]$
- ✓ $\lambda_l(\mathbf{M})$: the l-th eigenvalue of a matrix $\mathbf{M}$
- ✓ $\mathbf{f}_n$ is obtained thru exhaustive search over $\mathcal{F}_{RF}$

Wideband Hybrid Precoding System

(2) Gram-Schmidt greedy hybrid precoding

- ✓ $\mathbf{f}_n$ is chosen from the orthogonal complement of $\hat{\mathbf{F}}_{RF}^{(i-1)}$
- ✓ $\mathbf{f}_n$ can be obtained thru the Gram-Schmidt procedure

$$\max_{\mathbf{f}_n \in \mathcal{F}_{RF}} R^{(i)}(\mathbf{f}_n) = \sum_{k=1}^{K} \sum_{l=1}^{i} \log\left(1 + \frac{E_s}{K N_s N_0} \lambda_l \left(\mathbf{H}[k] \hat{\mathbf{F}}_{RF}^{(i,n)} \left(\hat{\mathbf{F}}_{RF}^{(i,n)H} \hat{\mathbf{F}}_{RF}^{(i,\boldsymbol{n})}\right)^{-1} \hat{\mathbf{F}}_{RF}^{(i,n)H} \mathbf{H}[k]^H\right)\right)$$

$$= \sum_{k=1}^{K} \sum_{l=1}^{i} \log\left(1 + \frac{E_s}{K N_s N_0} \lambda_l \left(\mathbf{T}^{(i-1)} + \mathbf{H}[k] \mathbf{f}_n \mathbf{f}_n^H \hat{\mathbf{F}}_{RF}^{(i,n)H} \mathbf{H}[k]^H\right)\right)$$

- ✓ $\mathbf{T}^{(i-1)} = \mathbf{H}[k] \hat{\mathbf{F}}_{RF}^{(i-1)} \left(\hat{\mathbf{F}}_{RF}^{(i-1)H} \hat{\mathbf{F}}_{RF}^{(i-1)}\right)^{-1} \hat{\mathbf{F}}_{RF}^{(i-1)H} \mathbf{H}[k]^H$
- ✓ The eigenvalues calculation is a rank-1 updateof the previous iteration eigenvalues

Wideband Hybrid Precoding System

(3) Approximate Gram-Schmidt based greedy hybrid precoding

- ✓ $\mathbf{f}_n$ is chosen from the orthogonal complement of $\hat{\mathbf{F}}_{RF}^{(i-1)}$

$$\max_{\mathbf{f}_n \in \mathcal{F}_{RF}} R^{(i)}(\mathbf{f}_n) = \sum_{k=1}^{K} \sum_{l=1}^{i} \log\left(1 + \frac{E_s}{K N_s N_0} \lambda_l \left(\mathbf{H}[k] \hat{\mathbf{F}}_{RF}^{(i,n)} \left(\hat{\mathbf{F}}_{RF}^{(i,n)H} \hat{\mathbf{F}}_{RF}^{(i,n)}\right)^{-1} \hat{\mathbf{F}}_{RF}^{(i,n)H} \mathbf{H}[k]^H\right)\right)$$

$$\approx \sum_{k=1}^{K} \left(\log\det\left(\mathbf{I} + \frac{E_s}{K N_s N_0} \widetilde{\boldsymbol{\Sigma}}[k]^2\right) - \mathrm{tr}\left(\widetilde{\boldsymbol{\Sigma}}[k]\right)\right) + \sum_{k=1}^{K} \left\| \widetilde{\boldsymbol{\Sigma}}[k] \widetilde{\mathbf{V}}[k]^H \hat{\mathbf{F}}_{RF}^{(i,n)} \left(\hat{\mathbf{F}}_{RF}^{(i,n)H} \hat{\mathbf{F}}_{RF}^{(i,n)}\right)^{-\frac{1}{2}} \right\|^2$$

- ✓ $\mathbf{H}[k] = \mathbf{U}[k] \boldsymbol{\Sigma}[k] \mathbf{V}[k]^H$
- ✓ $\widetilde{\boldsymbol{\Sigma}}[k] = [\boldsymbol{\Sigma}[k]]_{:,1:N_s}$
- ✓ $\widetilde{\mathbf{V}}[k] = [\mathbf{V}[k]]_{:,1:N_s}$

Wideband Hybrid Precoding System

(3) Approximate Gram-Schmidt based greedy hybrid precoding

- ✓ $\mathbf{f}_n$ can be optimized by

$$\mathbf{f}_n = \operatorname{argmax} \left\| \widetilde{\boldsymbol{\Sigma}}_{\boldsymbol{H}} \widetilde{\mathbf{V}}_{\boldsymbol{H}}{}^{H} \mathbf{f}_n \right\|^2$$

  - ✓ $\widetilde{\boldsymbol{\Sigma}}_{\boldsymbol{H}} = \left[\boldsymbol{\Sigma}[1], \dots, \boldsymbol{\Sigma}[K]\right]$
  - ✓ $\widetilde{\mathbf{V}}_{\boldsymbol{H}} = \left[\mathbf{V}[1], \dots, \mathbf{V}[K]\right]$
  - ✓ $\mathbf{f}_n \in \mathcal{F}_{RF}$ and $\mathbf{f}_n$ is orthogonal to the columns of $\hat{\mathbf{F}}_{RF}^{(i-1)}$

Algorithm 1 Approximate Gram-Schmidt Greedy Hybrid Precoding

Initialization

1) Construct $\mathbf{\Pi} = \tilde{\mathbf{\Sigma}}_{\mathbf{H}} \tilde{\mathbf{V}}_{\mathbf{H}}$, with $\tilde{\mathbf{\Sigma}}_{\mathbf{H}} = \left[\tilde{\mathbf{\Sigma}}_1, ..., \tilde{\mathbf{\Sigma}}_{\mathrm{K}}\right]$ and $\tilde{\mathbf{V}}_{\mathbf{H}} = \left[\tilde{\mathbf{V}}_1, ..., \tilde{\mathbf{V}}_{\mathrm{K}}\right]$. Set $\mathbf{F}_{\mathrm{RF}} =$ Empty Matrix. Set $\mathbf{A}_{\mathrm{CB}} = \left[\mathbf{f}_1^{\mathrm{RF}}, ..., \mathbf{f}_{N_{\mathrm{CB}}^{\mathrm{v}}}^{\mathrm{RF}}\right]$, where $\mathbf{f}_n^{\mathrm{RF}}, n = 1, ..., N_{\mathrm{CB}}^{\mathrm{v}}$ are the codewords in $\mathcal{F}_{\mathrm{RF}}$.

RF Precoder Design

2) For $i, i = 1, ..., N_{\mathrm{RF}}$

a) $\mathbf{\Psi} = \mathbf{\Pi}^* \mathbf{A}_{\mathrm{CB}}$

b) $n^\star = \arg\max_{n=1,2,..N_{\mathrm{CB}}^{\mathrm{v}}} \left\| [\mathbf{\Psi}]_{:,n} \right\|_2$.

c) $\mathbf{F}_{\mathrm{RF}}^{(i)} = \left[\mathbf{F}_{\mathrm{RF}}^{(i-1)} \mathbf{f}_{n^\star}^{\mathrm{RF}}\right]$

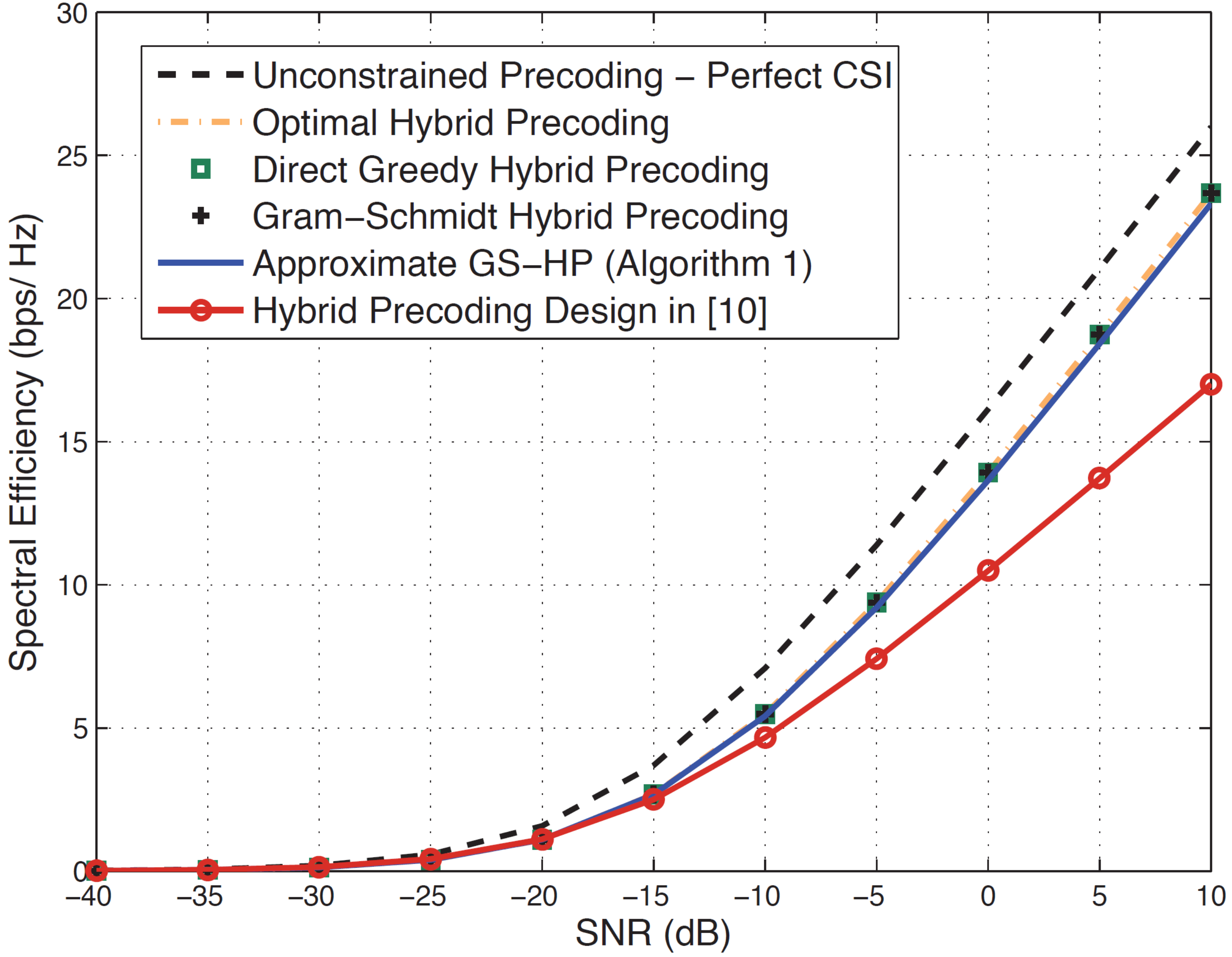
d) $\mathbf{\Pi} = \mathbf{\Pi} \left(\mathbf{I}_i - \mathbf{F}_{\mathrm{RF}}^{(i)} \left(\mathbf{F}_{\mathrm{RF}}^{(i)^*} \mathbf{F}_{\mathrm{RF}}^{(i)} \right)^{-1} \mathbf{F}_{\mathrm{RF}}^{(i)^*} \right)$

Digital Precoder Design

3) $\mathbf{F}[k] = \mathbf{F}_{\mathrm{RF}}^{(N_{\mathrm{RF}})} \left(\mathbf{F}_{\mathrm{RF}}^{(N_{\mathrm{RF}})^*} \mathbf{F}_{\mathrm{RF}}^{(N_{\mathrm{RF}})} \right)^{-\frac{1}{2}} \left[\overline{\mathbf{V}}[k]\right]_{:,1:N_{\mathrm{S}}}, k = 1, ..., K$, with $\overline{\mathbf{V}}[k]$ defined in (5).

Performance Comparison

- ✓ 32×16 MIMO
- ✓ $N_{RF} = 3$



- Massive MIMO and Hybrid Precoder/Combiner
- Narrowband Hybrid Precoder/Combiner Designs
- Wideband Hybrid Precoder/Combiner Designs
- **Remarks**

Remarks

➢ Hybrid Precoder/Combiner compromise the tradeoff between hardware cost and spectral efficiency

➢ We studied optimization algorithms of hybrid precoder:

1. Narrowband fully-connected hybrid precoder: OMP and MO based algorithms
2. Narrowband sub-connected hybrid precoder: SDR and SIC based algorithms
3. Wideband hybrid precoder: Gram-Schmidt greedy algorithms