From MIMO to Massive MIMO

Part I : Basics on Digital and Analog MIMO

2024/07/01

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- Introduction
- MIMO Channel Model
- Fully Digital Precoder & Combiner
- Hardware Consideration and Analog Precoder
- Remarks

Evolution of Mobile Communications

1G
- 1980's
- Analog
- 2kbps

2G
- 1990's
- TDMA
- Frequency Reuse
- 10-500 kbps

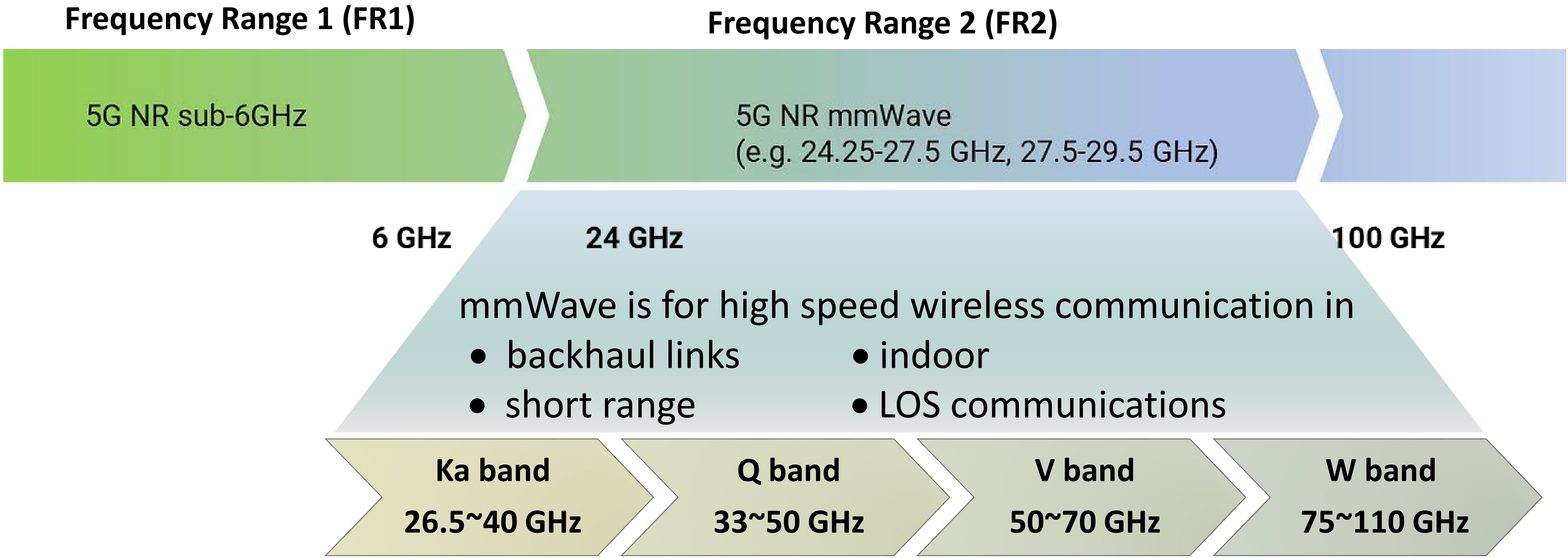
3G
- 2000's
- CDMA
- Broadband
- 400kbps-30Mbps

4G
- 2010's
- OFDMA
- **MIMO**
- CoMP
- 200Mbps-1Gbps

5G
- 2020's
- **Massive MIMO**
- Network Programmability
- 1-20Gbps

6G
- 2030's
- ?

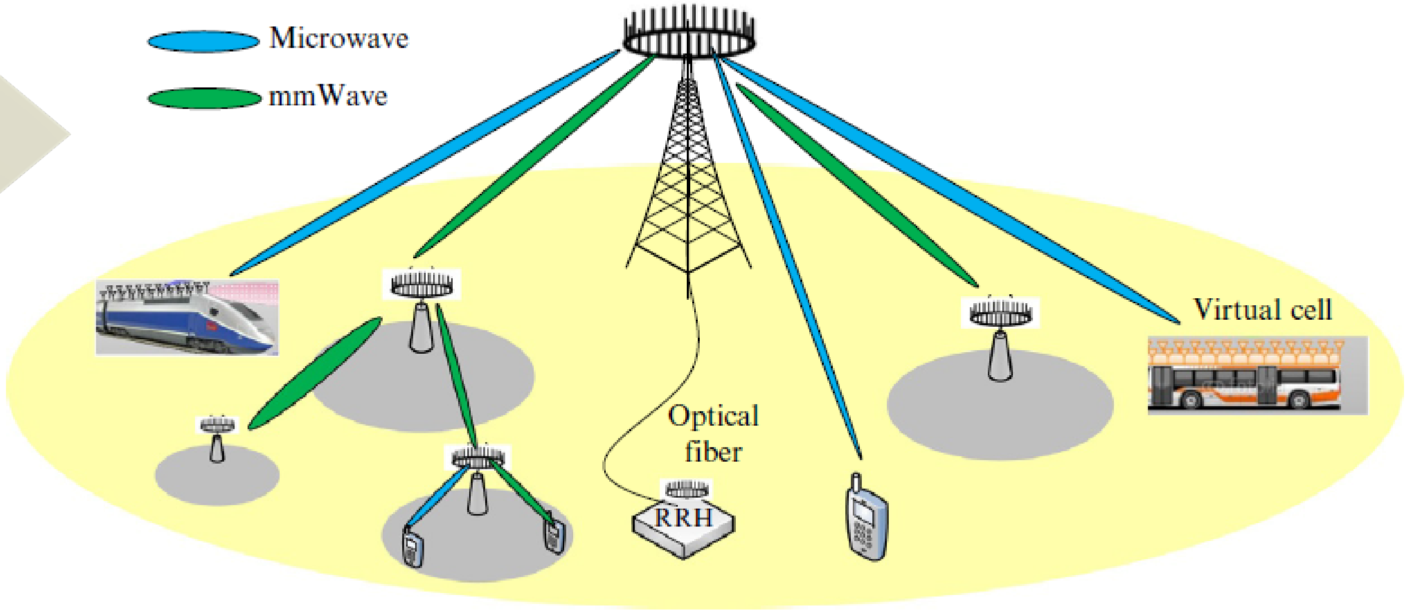
Spectrum of 5G NR



Challenges of mmWave Spectrum

- Propagation Loss
- Penetration Loss
- Rain Fading
- Absorbed or Scattered by Gases

Massive MIMO is required to deal with the challenges on the severe fading and attenuation

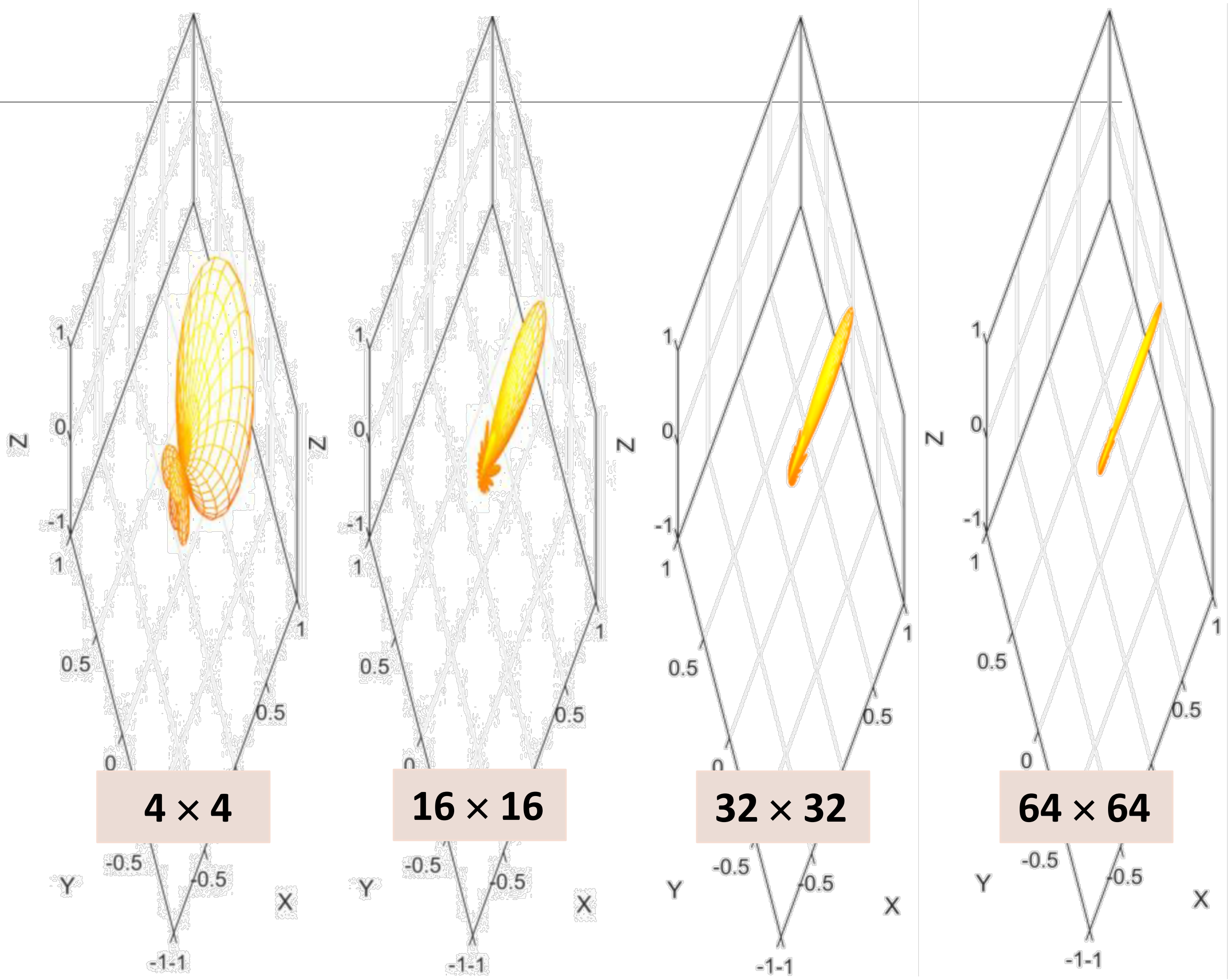


When there are more and more antennas

Beam-pattern of antenna array

As the antenna number increases:

- ✓ Narrower beam-width
- ✓ Stronger beam strength
- ✓ Higher array gain



100-Antenna Massive MIMO testbed

@Lund University, 2015

22User

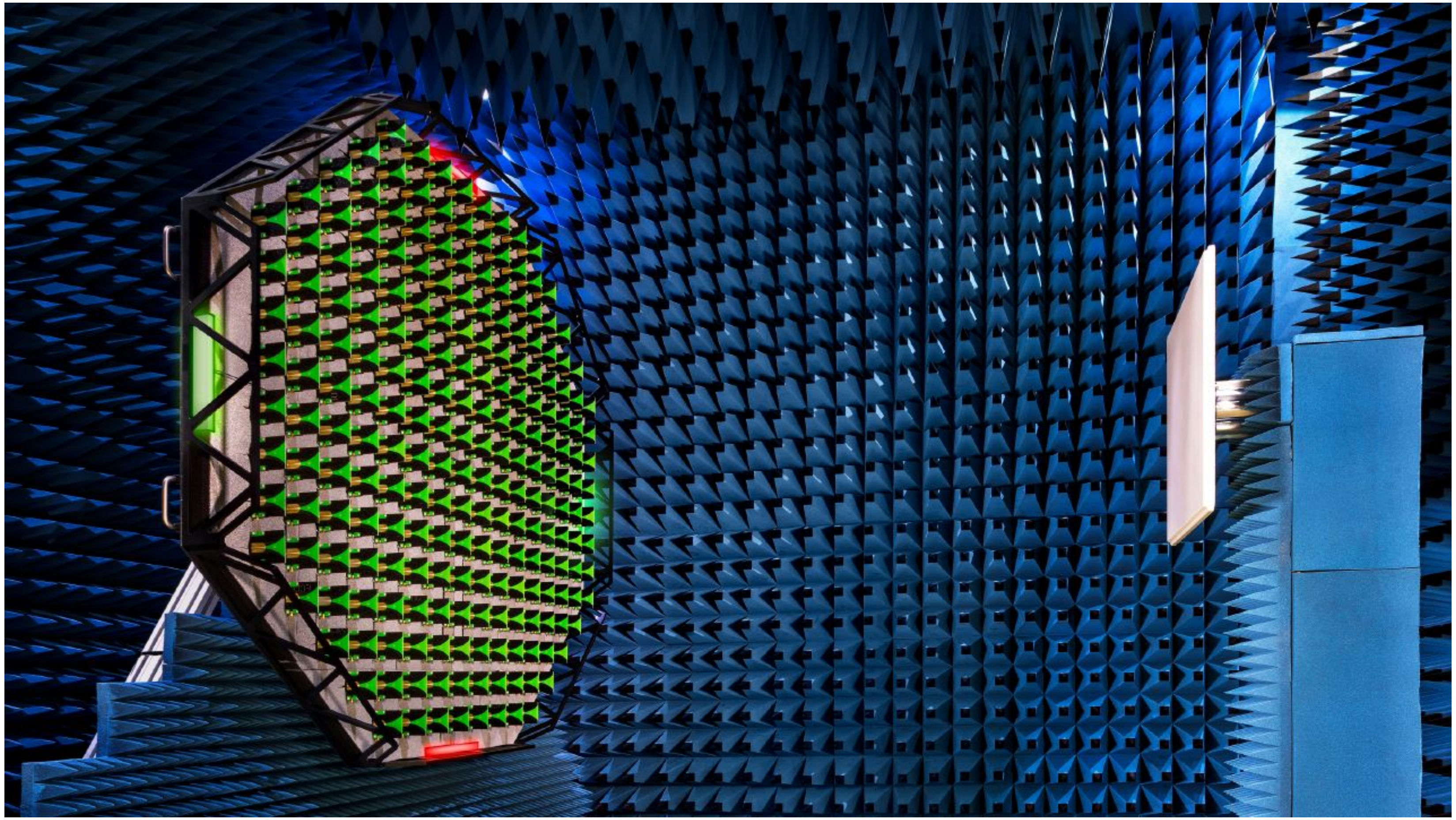
Spectral Efficiency 145.6bps/Hz

3.51GHz, BW=20MHz

SE greatly improved compared with 4G(3bps/Hz)

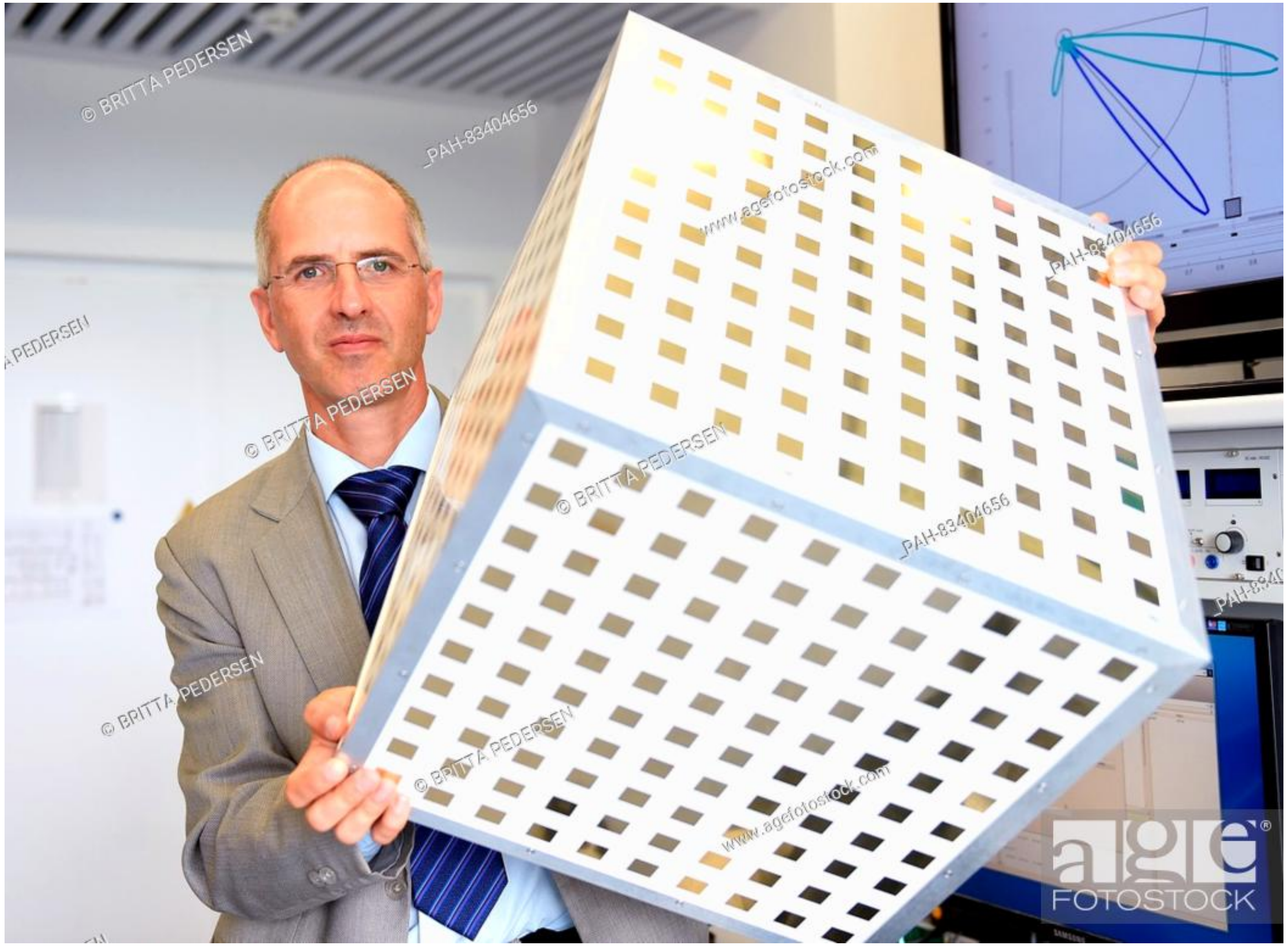
OTA Test of Massive MIMO Systems

Rohde & Schwarz, 2018



Antenna Array of Massive MIMO Systems

Fraunhofer HHI, Germany



Benefits of massive MIMO

Increase Network Capacity

- Higher spatial multiplexing gain
- More spatial streams sent simultaneously

Improve Coverage

- 3D beamforming enables dynamic coverage

Simplify the multiple-access layer

Improve energy efficiency

Reduce interference

Robust to intentional jamming

Challenges of Massive MIMO

High computational power for signal processing

- Multiuser detection
- Precoder optimization

Accurate channel estimation is tough

- Especially with insufficient RF chains
- Pilot contamination issue

High implementation cost

- Hardware impairment issue
- Synchronization in a large scale networks

Require Interference management

- User scheduling
- Beam management

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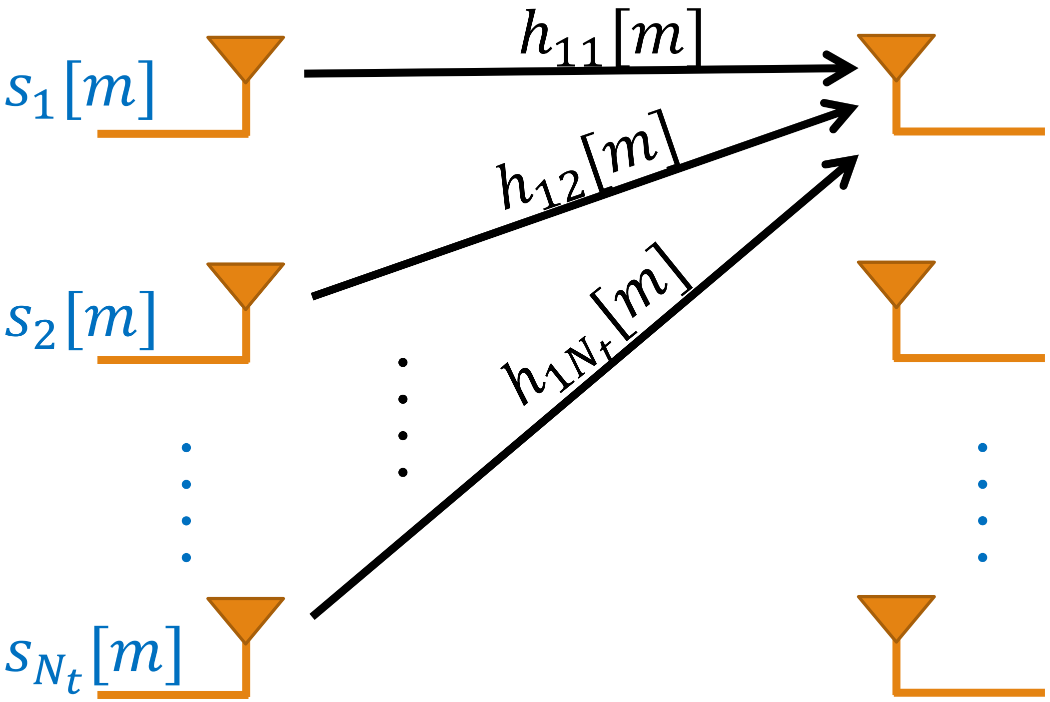
MIMO Channel

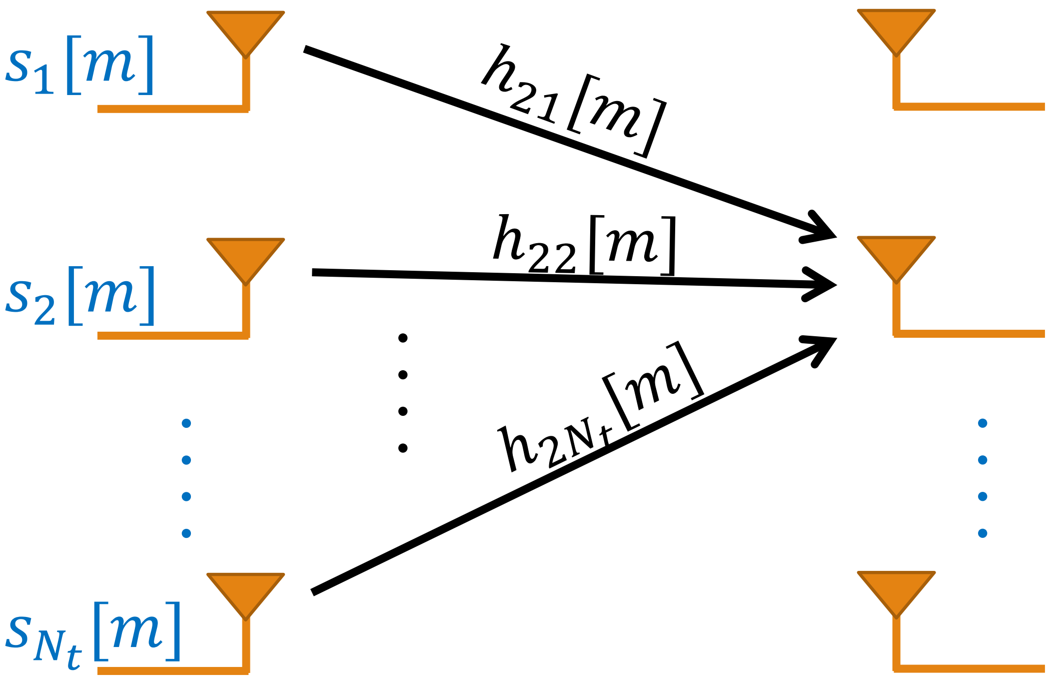
MIMO = Multiple Input Multiple Output



$$y_1[m] = \sum_{k=1}^{N_t} h_{1k}[m] s_k[m] + w_1[m]$$

MIMO Channel

MIMO = Multiple Input Multiple Output

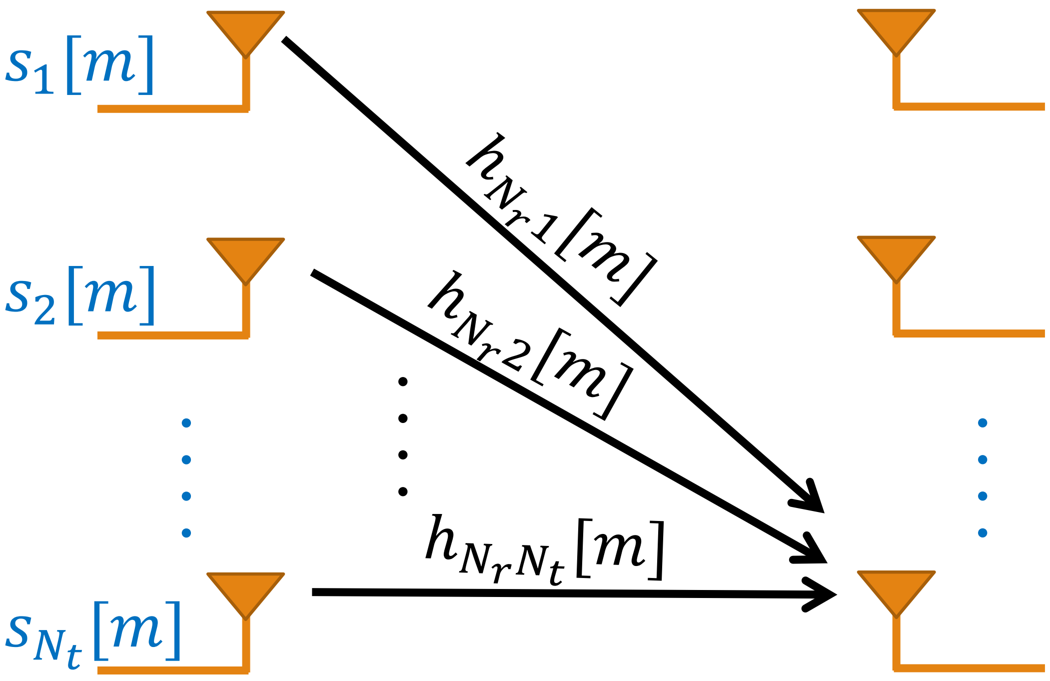


$$y_1[m] = \sum_{k=1}^{N_t} h_{1k}[m] s_k[m] + w_1[m]$$

$$y_2[m] = \sum_{k=1}^{N_t} h_{2k}[m] s_k[m] + w_2[m]$$

MIMO Channel

MIMO = Multiple Input Multiple Output



$$y_1[m] = \sum_{k=1}^{N_t} h_{1k}[m] s_k[m] + w_1[m]$$

$$y_2[m] = \sum_{k=1}^{N_t} h_{2k}[m] s_k[m] + w_2[m]$$

$$\vdots$$

$$y_{N_r}[m] = \sum_{k=1}^{N_t} h_{N_r k}[m] s_k[m] + w_{N_r}[m]$$

MIMO Channel

MIMO = Multiple Input Multiple Output

$$\underbrace{\begin{bmatrix} y_1[m] \\ y_2[m] \\ \vdots \\ y_{N_r}[m] \end{bmatrix}}_{\mathbf{y}[m]} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} s_1[m] \\ s_2[m] \\ \vdots \\ s_{N_t}[m] \end{bmatrix}}_{\mathbf{s}[m]} + \underbrace{\begin{bmatrix} w_1[m] \\ w_2[m] \\ \vdots \\ w_{N_r}[m] \end{bmatrix}}_{\mathbf{w}[m]}$$

MIMO Channel

$$\mathbf{y}[m] = \mathbf{H}\mathbf{s}[m] + \mathbf{w}[m]$$

- ✓ $\mathbf{s}[m]$ has zero-mean with covariance matrix

$$R_s = E[s[m]s[m]^H]$$

- ✓ $\mathbf{w}[m]$ has zero-mean with covariance matrix

$$R_w = E[w[m]w[m]^H] = N_0 I$$

- ✓ $\mathbf{s}[m]$ satisfies the power constraint :

$$E[\|s[m]\|^2] = E[|s_1[m]|^2] + E[|s_2[m]|^2] + \cdots + E[|s_{N_t}[m]|^2] \leq E_s$$

Channel Model

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_{LOS} + \sqrt{\frac{1}{K+1}}\mathbf{H}_{NLOS}$$

$\mathbf{H}_{LOS}$: line-of-sight (LOS) path

$\mathbf{H}_{NLOS}$: none line-of-sight (NLOS) path

K : Rician K factor

Channel Model – LOS Path

$$\mathbf{H}_{LOS} = \alpha_0 \boldsymbol{a}_{\boldsymbol{r}}(\phi_0)\boldsymbol{a}_{\boldsymbol{t}}^H(\theta_0)$$

$\boldsymbol{a}_{\boldsymbol{t}}, \boldsymbol{a}_{\boldsymbol{r}}$: array response vector:

$$\boldsymbol{a}_{\boldsymbol{t}}(\theta) = \frac{1}{\sqrt{N_t}}\left[1 \quad \exp(j\tfrac{2\pi}{\lambda}d_t \sin\theta) \quad \exp(j\tfrac{2\pi}{\lambda}2d_t \sin\theta) \quad \cdots \quad \exp(j\tfrac{2\pi}{\lambda}(N_t-1)d_t \sin\theta)\right]^{\boldsymbol{T}}$$

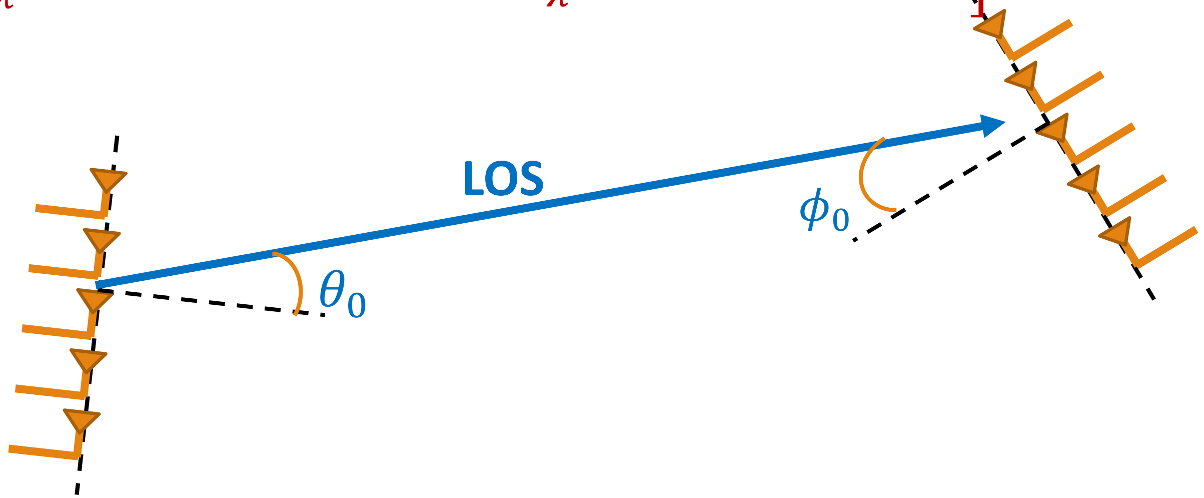
$$\boldsymbol{a}_{\boldsymbol{r}}(\phi) = \frac{1}{\sqrt{N_r}}\left[1 \quad \exp(j\tfrac{2\pi}{\lambda}d_r \sin\phi) \quad \exp(j\tfrac{2\pi}{\lambda}2d_r \sin\phi) \quad \cdots \quad \exp(j\tfrac{2\pi}{\lambda}(N_r-1)d_r \sin\phi)\right]^{\boldsymbol{T}}$$

θ_0 : angle of departure (AoD) for LOS path

ϕ_0 : angle of arrival (AoA) for LOS path

α_0 : complex gain of LOS path

$$\alpha_0 \sim CN(0, N_t N_r \cdot \mathrm{PL})$$



Channel Model – NLOS Paths

- For NLOS paths, there are several models depending on environments
- For rich scattering environments, there are numerous NLOS paths
- According to the CLM, $\mathbf{H}_{NLOS}$ can be expressed as

$$\mathbf{H}_{NLOS} = \boldsymbol{R}_r^{1/2}\mathbf{H}_{\text{iid}}\boldsymbol{R}_t^{1/2}$$

  - $\mathbf{H}_{\text{iid}}$: $N_r \times N_t$ matrix where each entry is i.i.d. with $CN(0, \mathrm{PL})$
  - $\mathbf{R}_t$: $N_t \times N_t$ correlation matrix among transmit antennas
  - $\mathbf{R}_r$: $N_r \times N_r$ correlation matrix among receive antennas
  - If antenna spacing are all greater than $\lambda/2$, $\mathbf{R}_t = \mathbf{R}_r = \mathbf{I}$

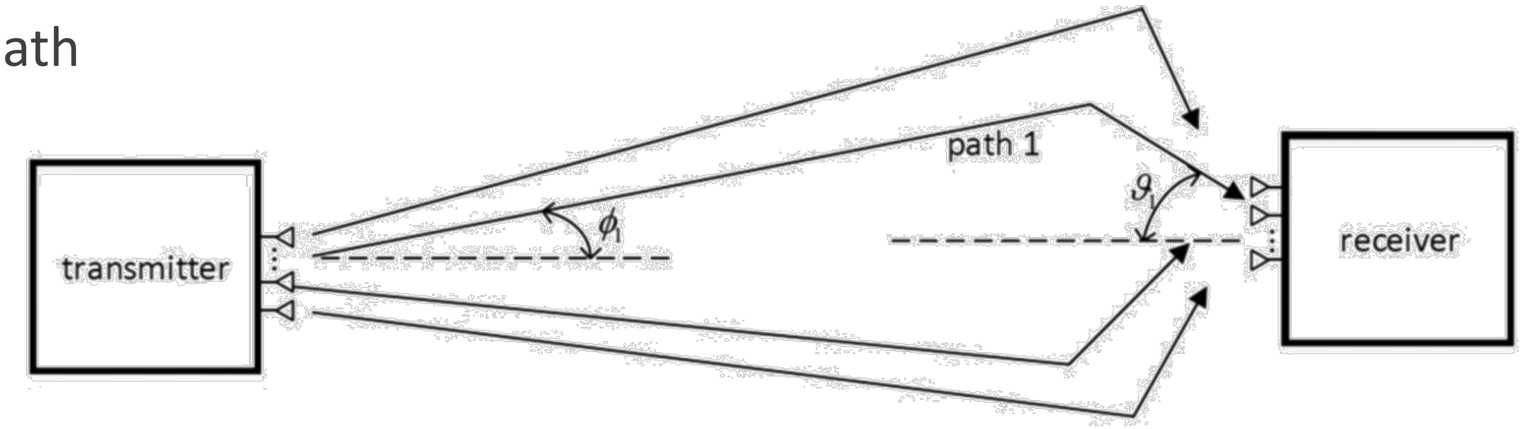
Channel Model – NLOS Paths

Under mmWave propagation, the number of paths is limited

$$\mathbf{H}_{NLOS} = \sum_{\ell=1}^{L} \alpha_\ell \boldsymbol{a}_{\boldsymbol{r}}(\phi_\ell) \boldsymbol{a}_{\boldsymbol{t}}^{H}(\theta_\ell)$$

- L : Number of the NLOSpaths
- θ_ℓ : AoDfor the ℓ–th NLOS path
- ϕ_ℓ : AoA for the ℓ–th NLOS path
- α_ℓ : complex gain of
  the ℓ–th NLOS path

$$\alpha_\ell \sim CN(0, \frac{N_t N_r}{L} \cdot \mathrm{PL})$$



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MIMO Channel

- Consider the MIMO channel

$$\mathbf{y}[m] = \mathbf{H}\mathbf{s}[m] + \mathbf{w}[m]$$

  - ✓ Assume that channel $\mathbf{H}$ is known at tx and rx

- Take singular value decomposition (SVD) on $\mathbf{H}$

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$$

  - ✓ $\mathbf{U}$: $N_r \times N_r$ unitary matrix
  - ✓ $\mathbf{V}$: $N_t \times N_t$ unitary matrix
  - ✓ $\boldsymbol{\Sigma}$: $N_r \times N_t$ diagonal matrix with descending diagonal entries

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0,0,\ldots,0$$

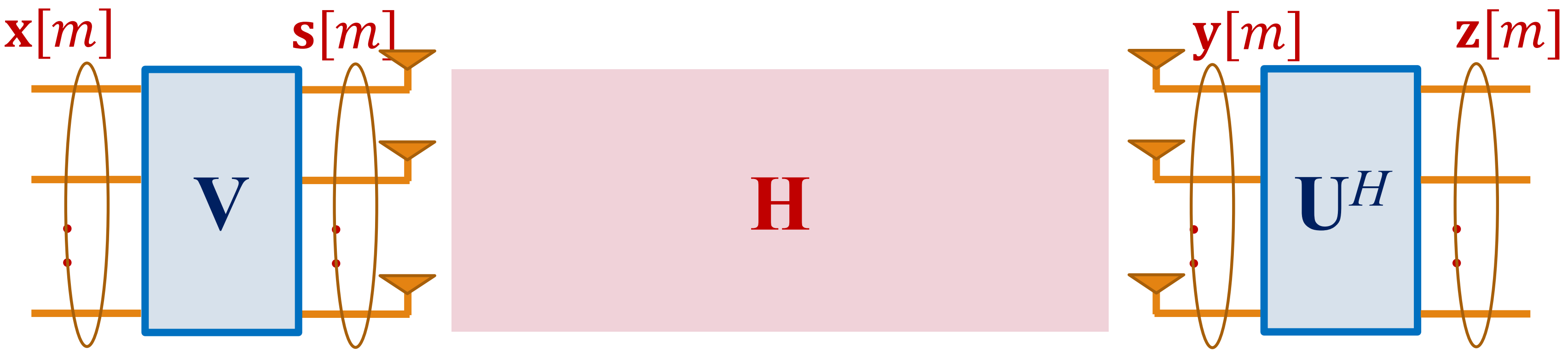
  - ✓ $r = rank(\mathbf{H})$, usually, $\mathbf{H}$ is full-rank and $r = \min(N_t, N_r)$

Fully Digital Precoding Design

- Let $\mathbf{s}[m] = \mathbf{V}\mathbf{x}[m]$, $\mathbf{V}$ is the precoding matrix
  $\mathbf{z}[m] = \mathbf{U}^H\mathbf{y}[m]$, $\mathbf{U}^H$ is the decoding matrix

$$\begin{aligned}\mathbf{z}[m] &= \mathbf{U}^H\mathbf{y}[m] \\ &= \mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H\mathbf{V}\mathbf{x}[m] + \mathbf{U}^H\mathbf{w}[m] \\ &= \boldsymbol{\Sigma}\mathbf{x}[m] + \mathbf{U}^H\mathbf{w}[m]\end{aligned}$$

where $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \dots, \sigma_r)$

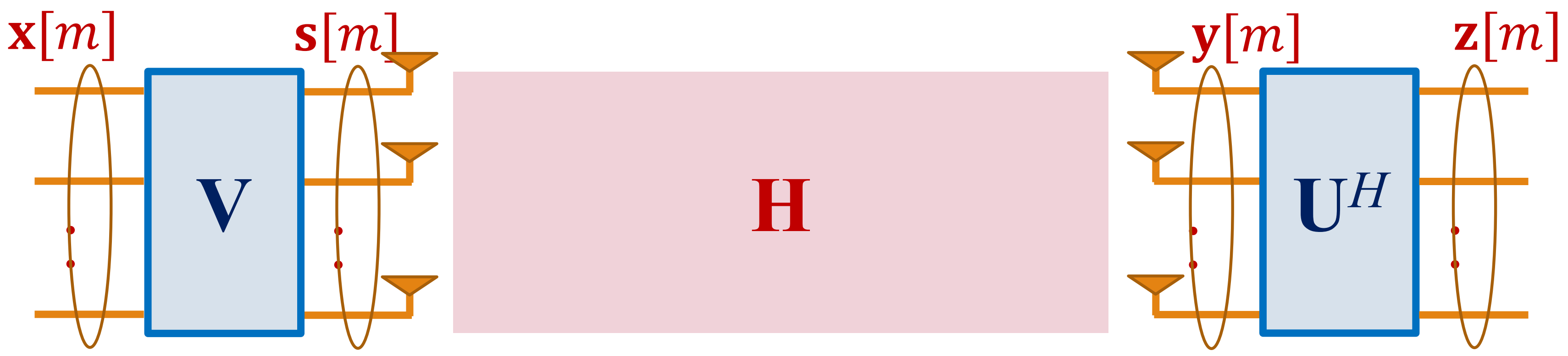


Fully Digital Precoding Design

Using SVD of $\mathbf{H}$: $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$

$$\mathbf{z}[m] = \boldsymbol{\Sigma}\mathbf{x}[m] + \tilde{\mathbf{w}}[m]$$

- ✓ $\tilde{\mathbf{w}}[m] = \mathbf{U}^H\mathbf{w}[m] \sim CN(\mathbf{0}, N_0\mathbf{I})$
- ✓ $\|\mathbf{x}[m]\|^2 = \|\mathbf{s}[m]\|^2 \Longrightarrow tr(\mathbf{R}_\mathbf{x}) \leq E_s$



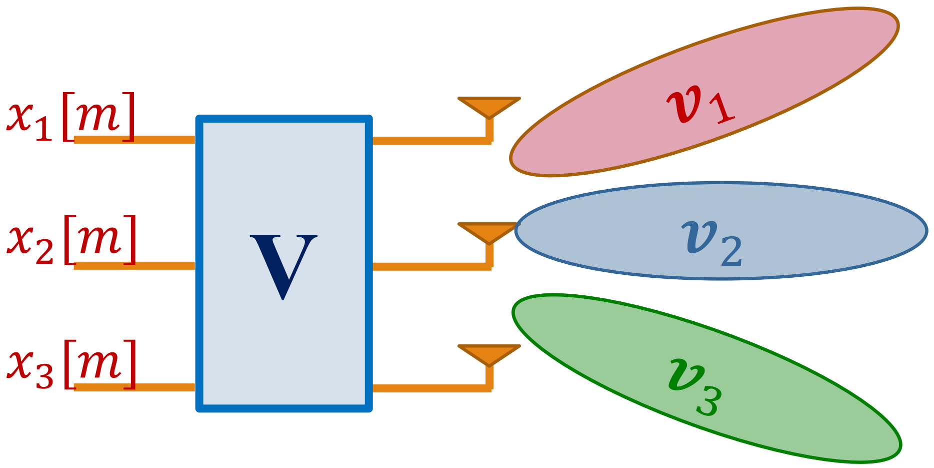
Fully Digital Precoding Design for 3x5 MIMO

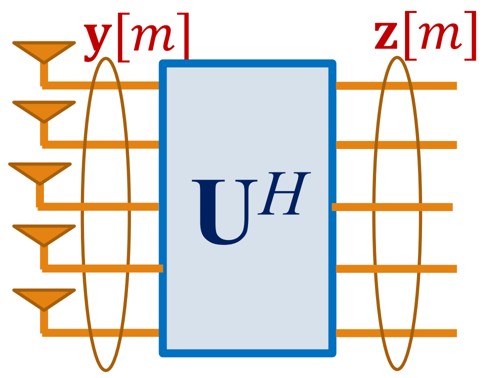
(ex) $N_t = 3, N_r = 5$

$\because \mathbf{s}[m] = \mathbf{V}\mathbf{x}[m] = \boldsymbol{v}_1 \cdot x_1[m] + \boldsymbol{v}_2 \cdot x_2[m] + \boldsymbol{v}_3 \cdot x_3[m]$

Where $\mathbf{V} = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3]$

It is equivalent to precode $x_i[m]$ using the beam $\boldsymbol{v}_i$, $i = 1,2,3$





Fully Digital Precoding Design for 3x5 MIMO

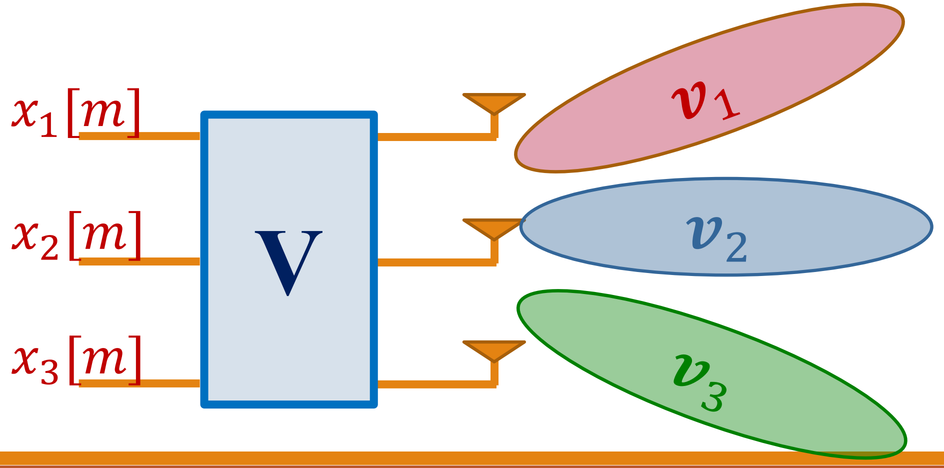
From the SVD of $\mathbf{H}$: $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H = \sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^H + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^H + \sigma_3 \boldsymbol{u}_3 \boldsymbol{v}_3^H$

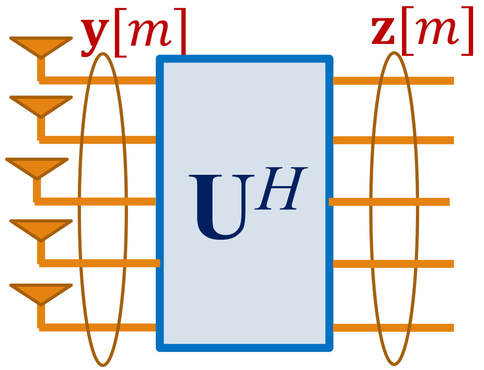
Since $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are mutually orthogonal

$$\mathbf{H}\boldsymbol{v}_1 = (\sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^H + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^H + \sigma_3 \boldsymbol{u}_3 \boldsymbol{v}_3^H)\, \boldsymbol{v}_1 = \sigma_1 \boldsymbol{u}_1$$

$$\mathbf{H}\boldsymbol{v}_2 = (\sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^H + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^H + \sigma_3 \boldsymbol{u}_3 \boldsymbol{v}_3^H)\, \boldsymbol{v}_2 = \sigma_2 \boldsymbol{u}_2$$

$$\mathbf{H}\boldsymbol{v}_3 = (\sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^H + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^H + \sigma_3 \boldsymbol{u}_3 \boldsymbol{v}_3^H)\, \boldsymbol{v}_3 = \sigma_3 \boldsymbol{u}_3$$





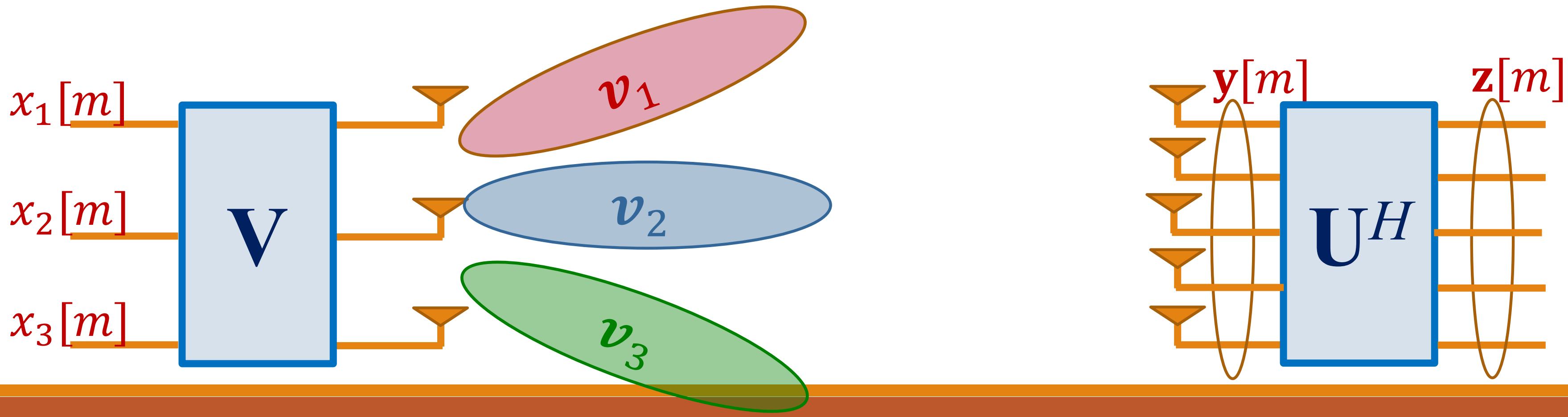
Fully Digital Precoding Design for 3x5 MIMO

$$\therefore \mathbf{y}[m] = \mathbf{Hs}[m] + \mathbf{w}[m]$$

$$= \mathbf{H}(\boldsymbol{v}_1 \cdot x_1[m] + \boldsymbol{v}_2 \cdot x_2[m] + \boldsymbol{v}_3 \cdot x_3[m]) + \mathbf{w}[m]$$

$$= \sigma_1 \boldsymbol{u}_1 \cdot x_1[m] + \sigma_2 \boldsymbol{u}_2 \cdot x_2[m] + \sigma_3 \boldsymbol{u}_3 \cdot x_3[m] + \mathbf{w}[m]$$

In $\mathbf{y}[m]$, $x_i[m]$ is weighted by the vector $\boldsymbol{u}_i$, $i = 1,2,3$



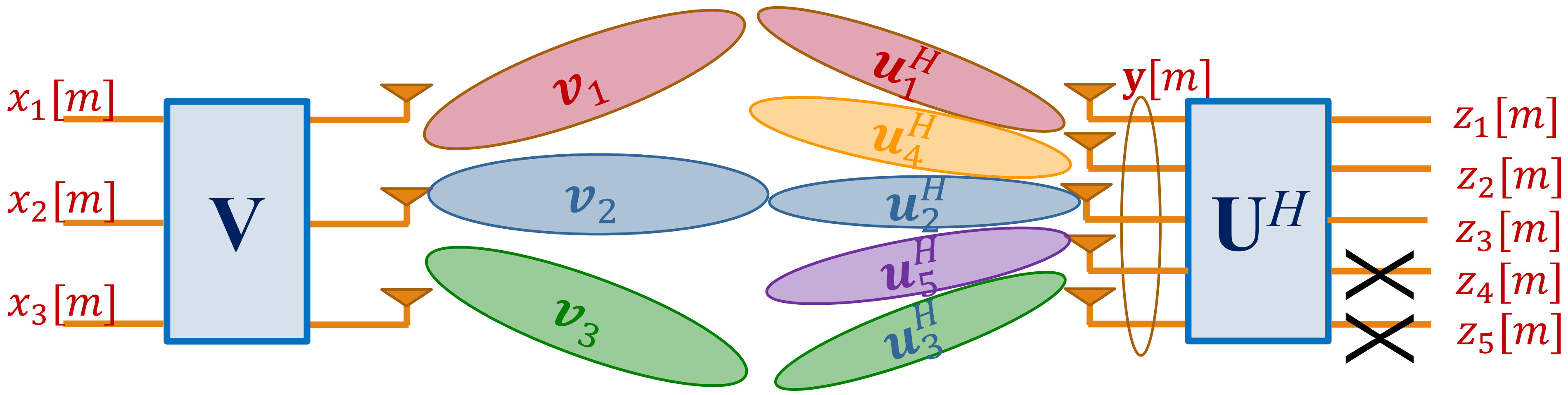
Fully Digital Precoding Design for 3x5 MIMO

At rx, Decoder is applied :

$\mathbf{z}[m] = \mathbf{U}^H \mathbf{y}[m]$ $\Longleftrightarrow$

$$\begin{bmatrix} z_1[m] \\ z_2[m] \\ z_3[m] \\ \cancel{z_4[m]} \\ \cancel{z_5[m]} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_1^H \mathbf{y}[m] \\ \boldsymbol{u}_2^H \mathbf{y}[m] \\ \boldsymbol{u}_3^H \mathbf{y}[m] \\ \cancel{\boldsymbol{u}_4^H \mathbf{y}[m]} \\ \cancel{\boldsymbol{u}_5^H \mathbf{y}[m]} \end{bmatrix} = \begin{bmatrix} \sigma_1 x_1[m] \\ \sigma_2 x_2[m] \\ \sigma_3 x_3[m] \\ 0 \\ 0 \end{bmatrix} + \mathbf{U}^H \mathbf{w}[m]$$

The last two output is removed since no data is conveyed



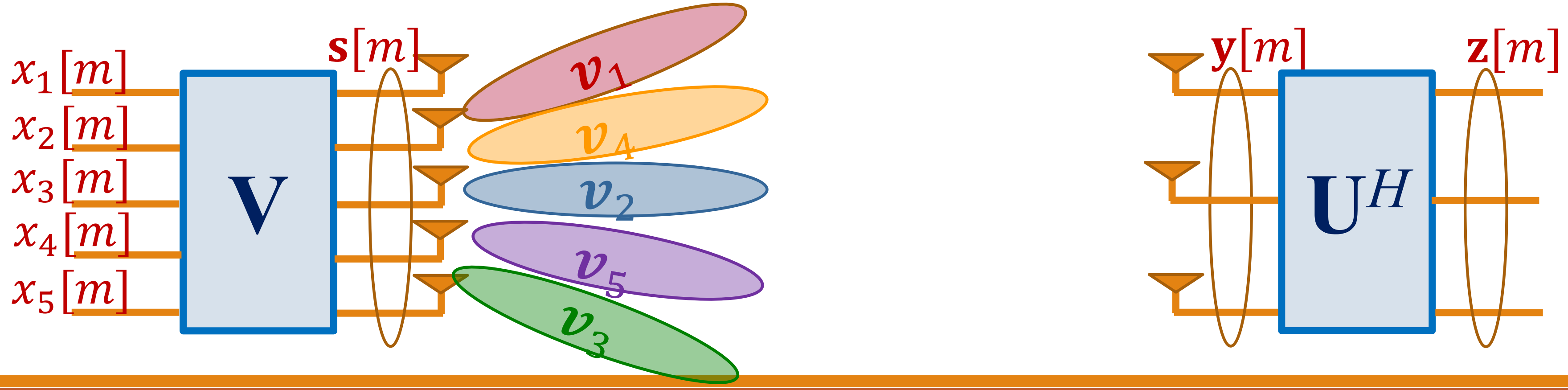
Fully Digital Precoding Design for 5x3 MIMO

(ex) $N_t = 5, N_r = 3$

$\because \mathbf{s}[m] = \mathbf{V}\mathbf{x}[m] = \boldsymbol{v}_1 \cdot x_1[m] + \boldsymbol{v}_2 \cdot x_2[m] + \boldsymbol{v}_3 \cdot x_3[m] + \boldsymbol{v}_4 \cdot x_4[m] + \boldsymbol{v}_5 \cdot x_5[m]$

where $\mathbf{V} = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \boldsymbol{v}_3 \quad \boldsymbol{v}_4 \quad \boldsymbol{v}_5]$

It is equivalent to precode $x_i[m]$ using the beam $\boldsymbol{v}_i$, $i = 1,2,3$



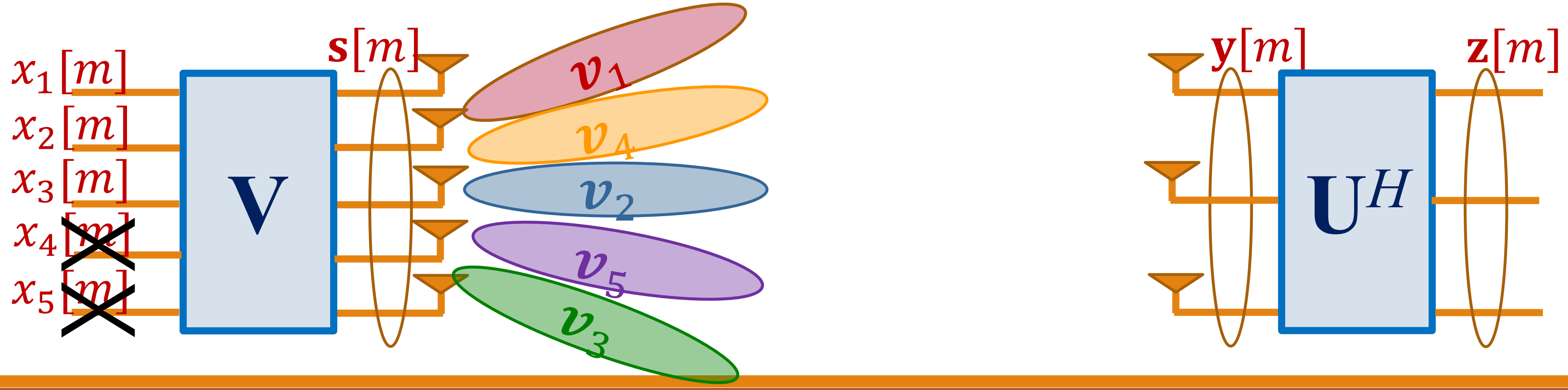
Fully Digital Precoding Design for 5x3 MIMO

From the SVD of $\mathbf{H}$: $\mathbf{H} = \mathbf{U\Sigma V}^H = \sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^H + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^H + \sigma_3 \boldsymbol{u}_3 \boldsymbol{v}_3^H$

$$\therefore \mathbf{y}[m] = \mathbf{Hs}[m] + \mathbf{w}[m]$$

$$= \sigma_1 \boldsymbol{u}_1 \cdot x_1[m] + \sigma_2 \boldsymbol{u}_2 \cdot x_2[m] + \sigma_3 \boldsymbol{u}_3 \cdot x_3[m] + \mathbf{w}[m]$$

The last two symbols, $x_4[m]$ and $x_5[m]$, cannot be delivered, so they are removed.

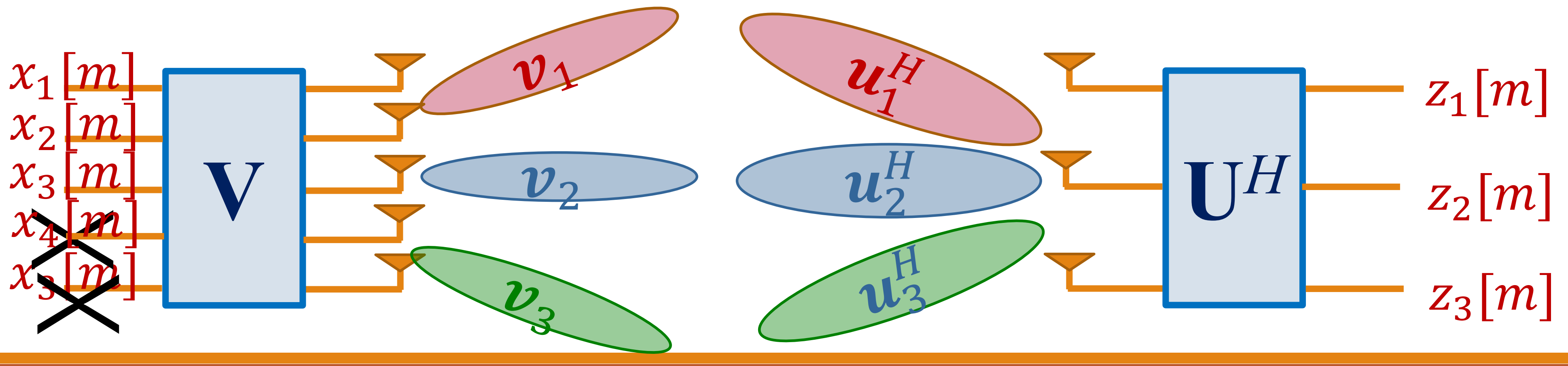


Fully Digital Precoding Design for 5x3 MIMO

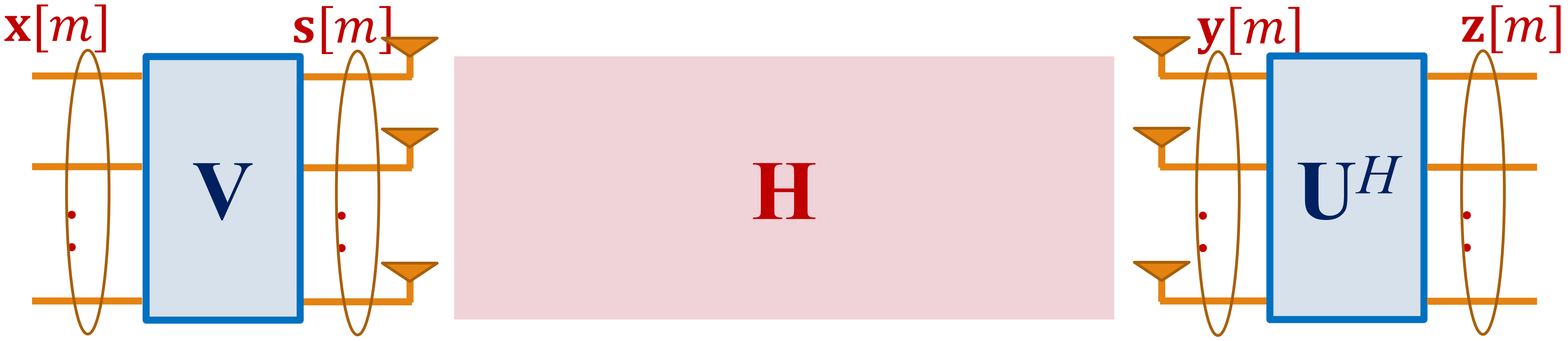
In $\mathbf{y}[m]$, $x_i[m]$ is weighted by the vector $\boldsymbol{u}_i$, $i = 1,2,3$

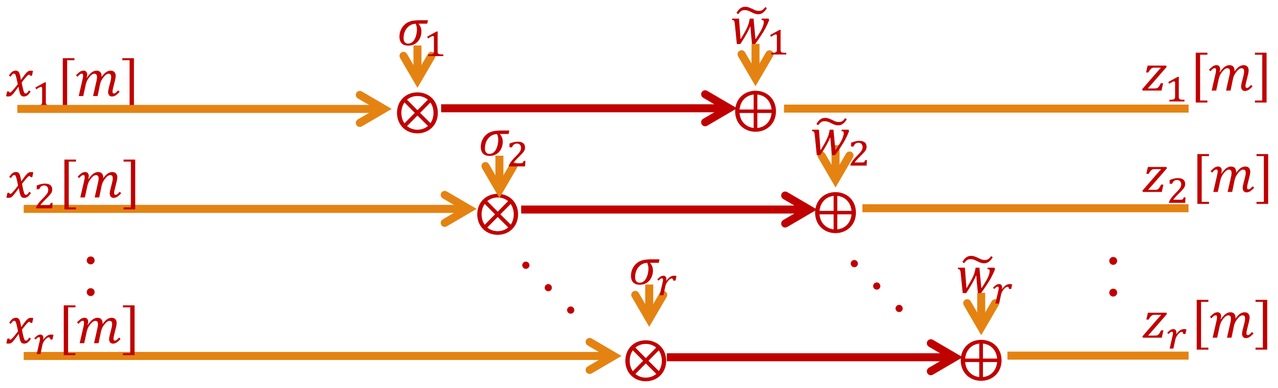
At rx, Decoder is applied :

$$\mathbf{z}[m] = \mathbf{U}^H \mathbf{y}[m] \Leftrightarrow \begin{bmatrix} z_1[m] \\ z_2[m] \\ z_3[m] \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_1^H \mathbf{y}[m] \\ \boldsymbol{u}_2^H \mathbf{y}[m] \\ \boldsymbol{u}_3^H \mathbf{y}[m] \end{bmatrix} = \begin{bmatrix} \sigma_1 x_1[m] \\ \sigma_2 x_2[m] \\ \sigma_3 x_3[m] \end{bmatrix} + \mathbf{U}^H \mathbf{w}[m]$$



Fully Digital Precoding Design



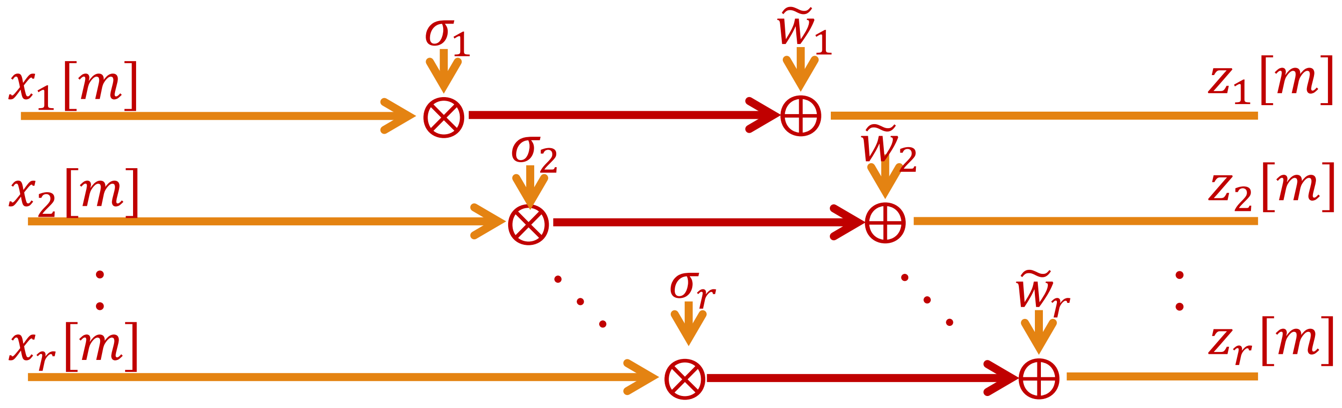


Fully Digital Precoding Design

$z_i[m] = \sigma_i x_i[m] + \tilde{w}_i[m]$, for $i = 1,2,\dots,r = rank(\mathbf{H})$

- ✓ To enhance reliability, set $x_i[m] = x[m],\ \forall i$
- ✓ To achieve highest rate, send indep. data on each channel

$$\mathbf{R_x} = \mathrm{diag}(E_1, E_2, \dots, E_r)$$



Capacity of MIMO Channel – Perfect CSIT

- With full CSI at tx, $\{E_i\}$ can be designed to maximize achievable rate of the MIMO channel

$$\max_{\{E_i\}} \sum_{i=1}^{r} \log\left(1 + \frac{E_i}{N_0}\sigma_i^2\right)$$

  - ✓ $\{E_i\}$ satisfy the power constraint :

$$\mathrm{tr}(\mathbf{R}_{\mathbf{x}}) = E_1 + E_2 + \cdots + E_r \le E_s$$

  - ✓ Use the method of Lagrange multiplier find the solution

- The optimal power allocation is :

$$E_i^* = \left(\mu - \frac{N_0}{\sigma_i^2}\right)^+$$

  - ✓ $(x)^+ \triangleq \max(x, 0)$
  - ✓ The value of μ is set to satisfies the power constraint :

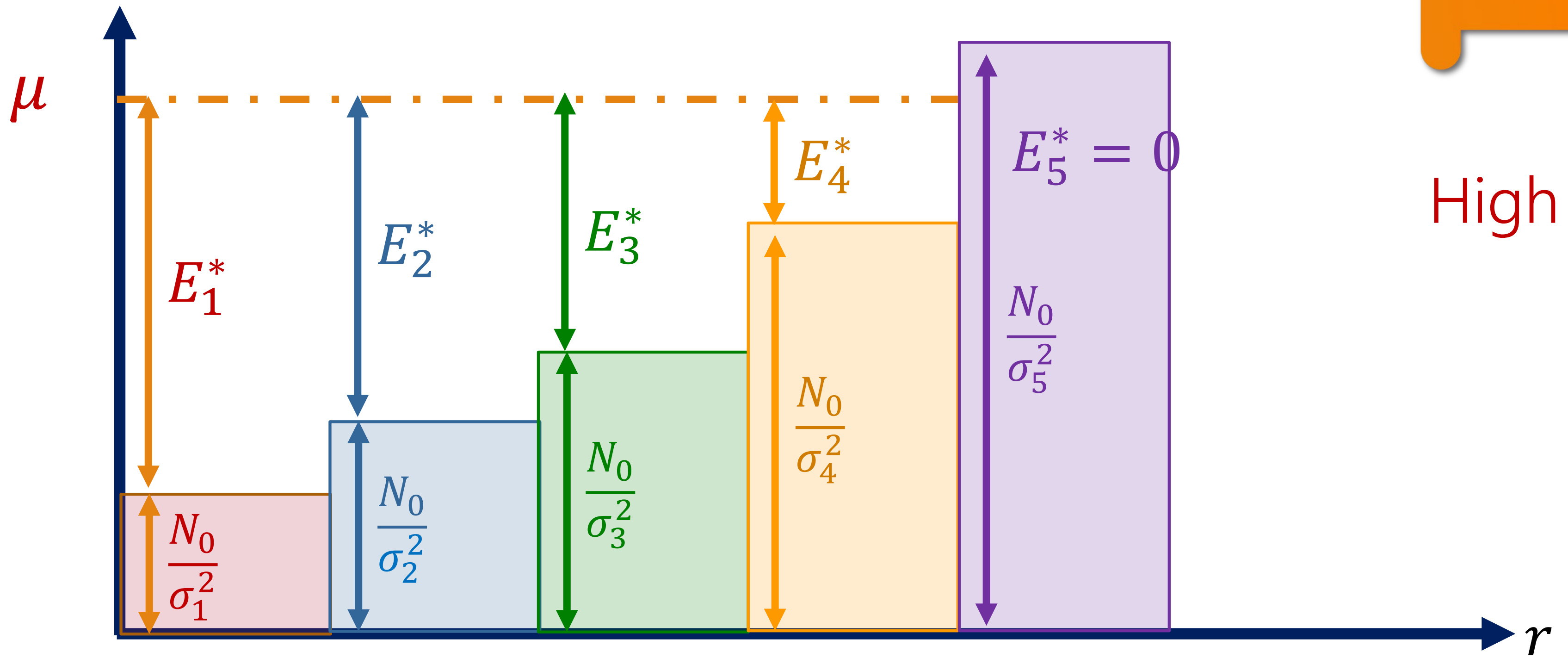
$$E_1^* + E_2^* + \cdots + E_r^* \le E_s$$

Capacity of MIMO Channel – Perfect CSIT

The optimal power allocation is :

$$E_i^* = \left(\mu - \frac{N_0}{\sigma_i^2}\right)^+$$

Water-filling Solution



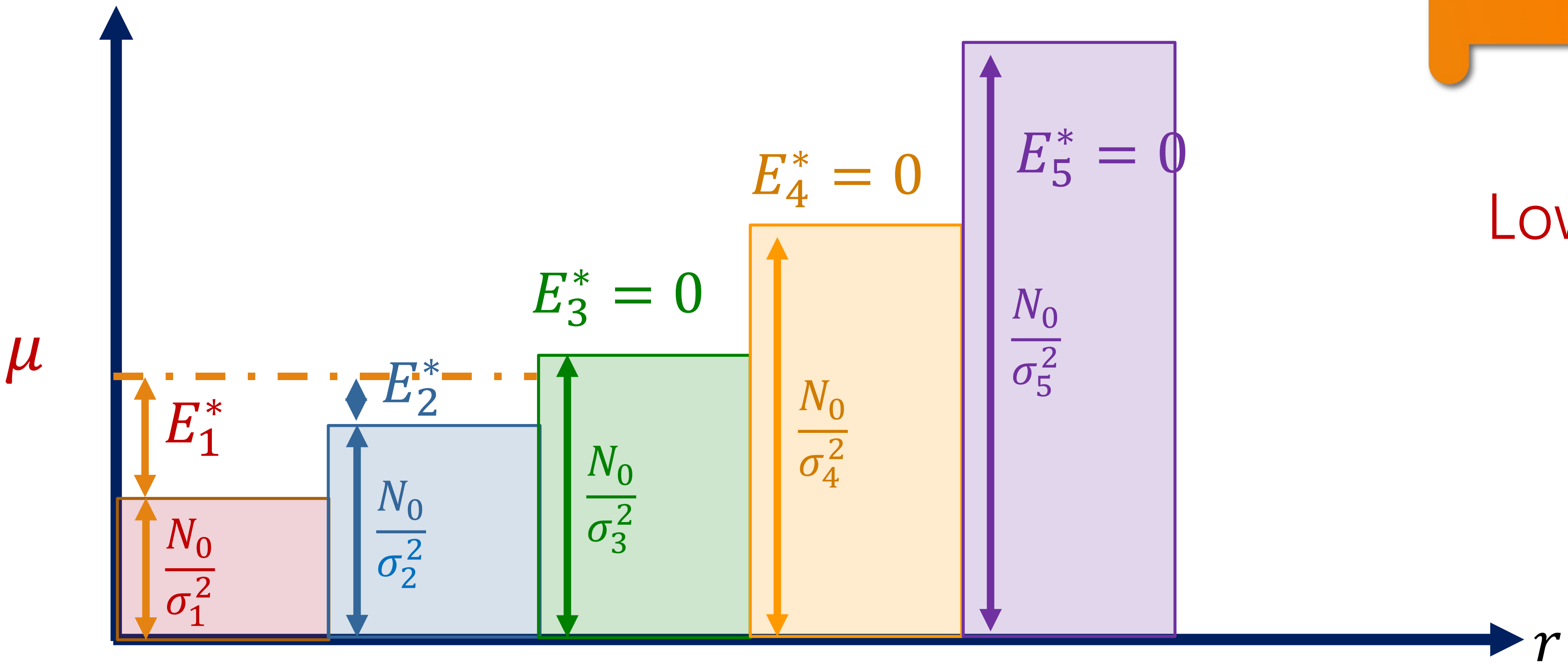
High SNR Case

Capacity of MIMO Channel – Perfect CSIT

The optimal power allocation is :

$$E_i^* = \left(\mu - \frac{N_0}{\sigma_i^2}\right)^+$$

Water-filling Solution



Low SNR Case

Capacity of MIMO Channel – Perfect CSIT

The optimal power allocation is :

$$E_i^* = \left(\mu - \frac{N_0}{\sigma_i^2}\right)^+$$

- ✓ Water-filling solution
- ✓ At low SNR, it tends to allocate all power to the best channel
- ✓ At high SNR, it tends to evenly allocate power

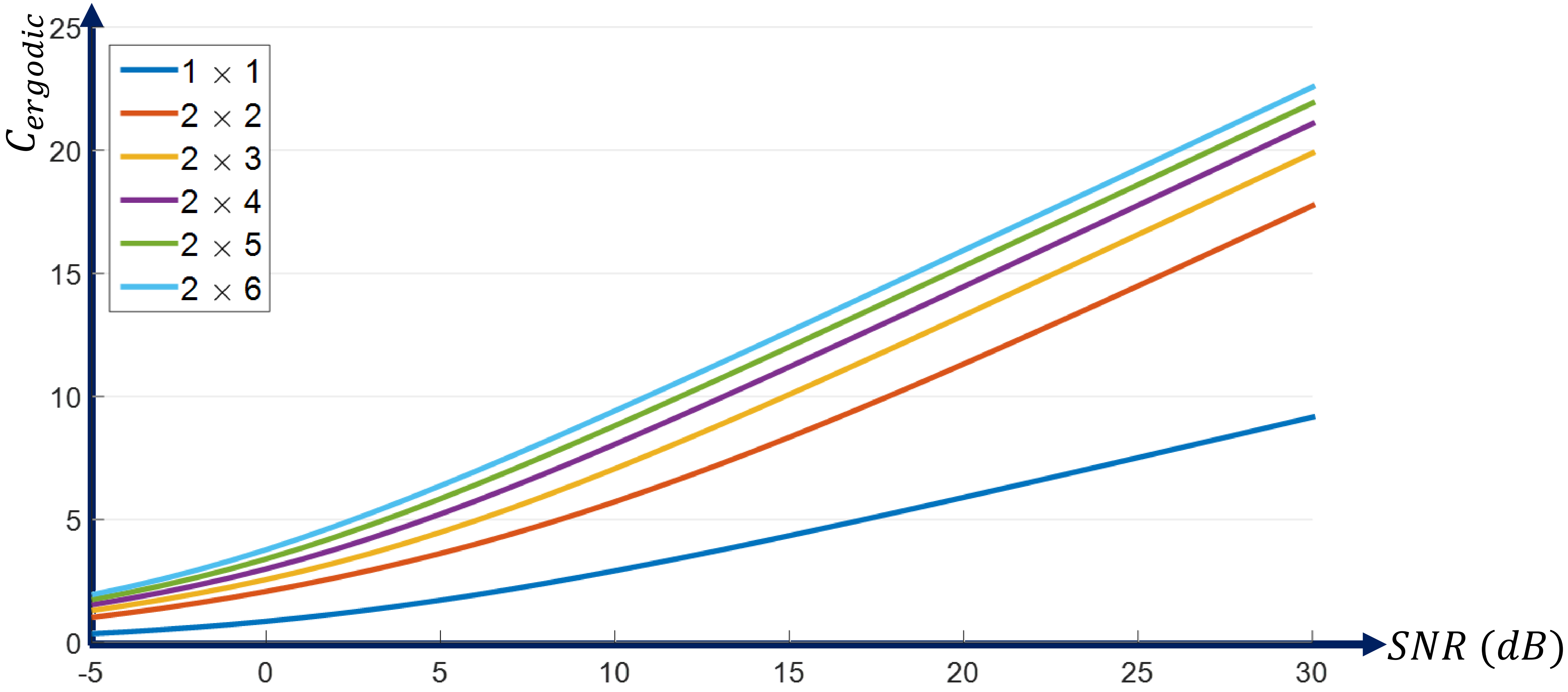
With CSI at tx, capacity of MIMO channel is

$$C = \sum_{i=1}^{r} \log\left(1 + \frac{E_i^*}{N_0}\sigma_i^2\right)$$

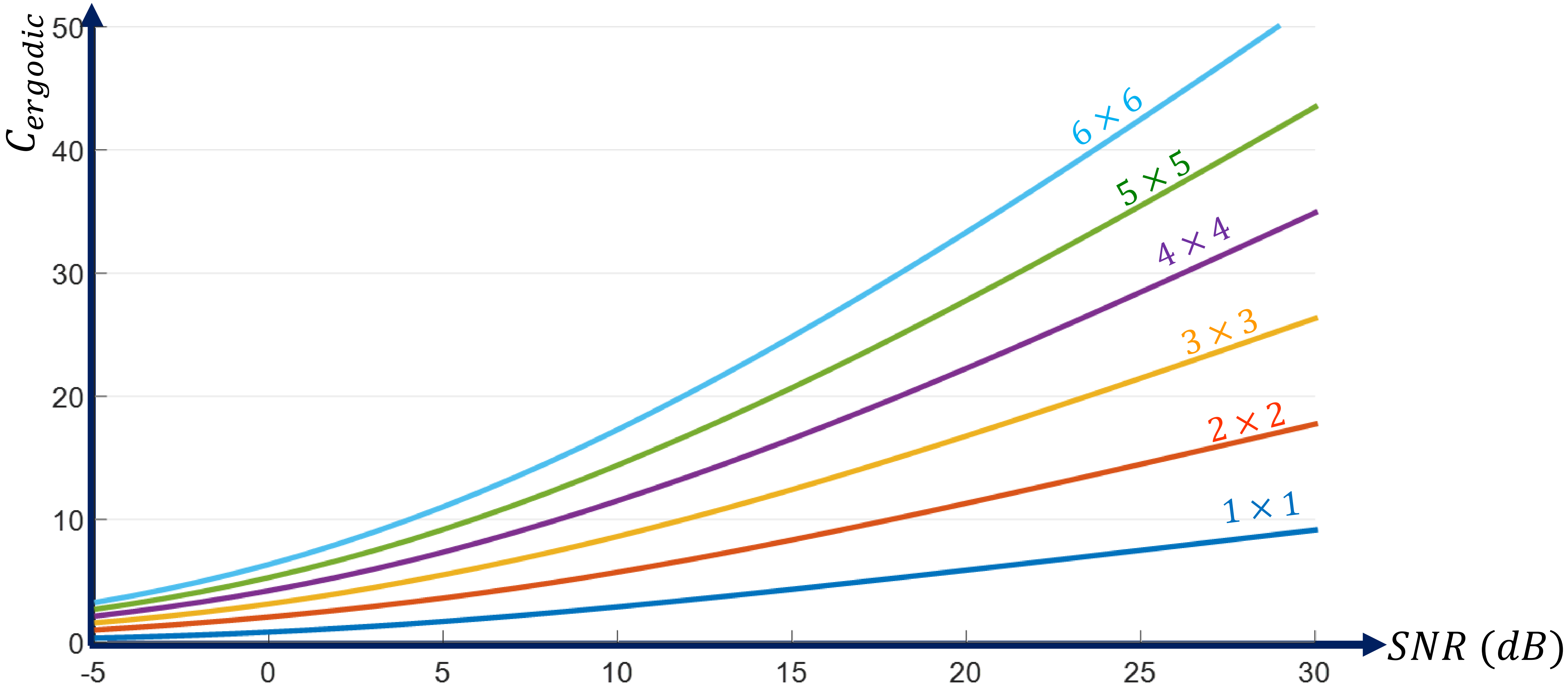
- ✓ The value of μ is set to satisfies the power constraint :

$$E_1^* + E_2^* + \cdots + E_r^* \leq E_s$$

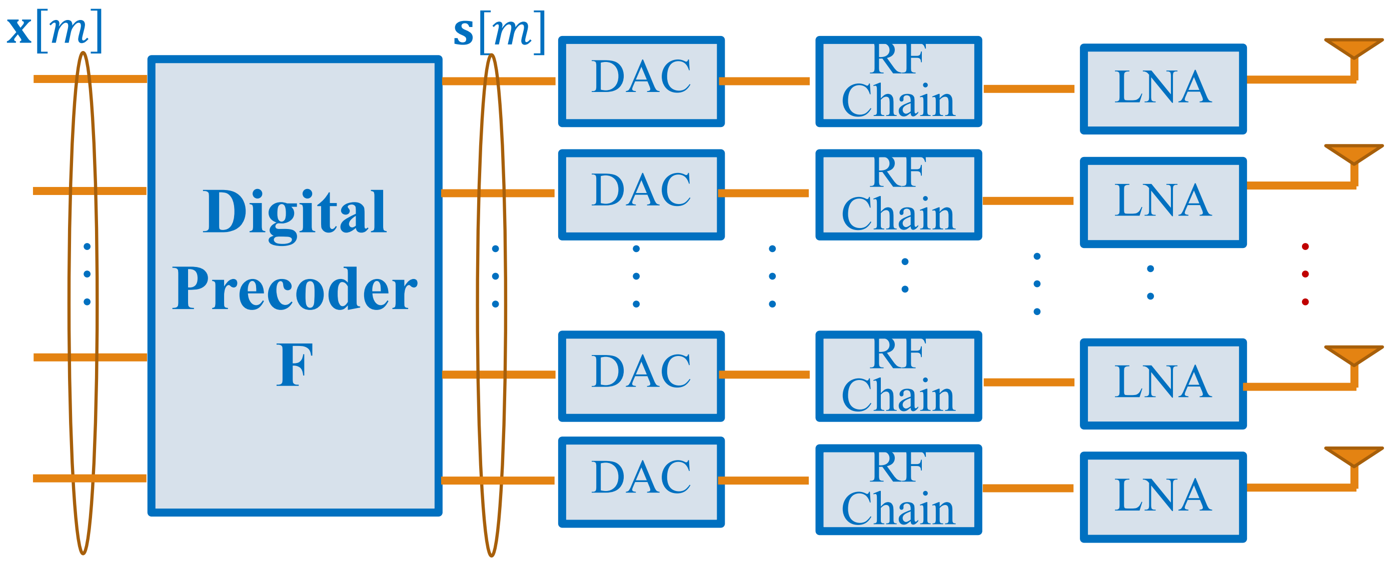
Capacity of MIMO Channel – Perfect CSIT



Capacity of MIMO Channel – Perfect CSIT



Fully Digital Precoder at MIMO Transmitter



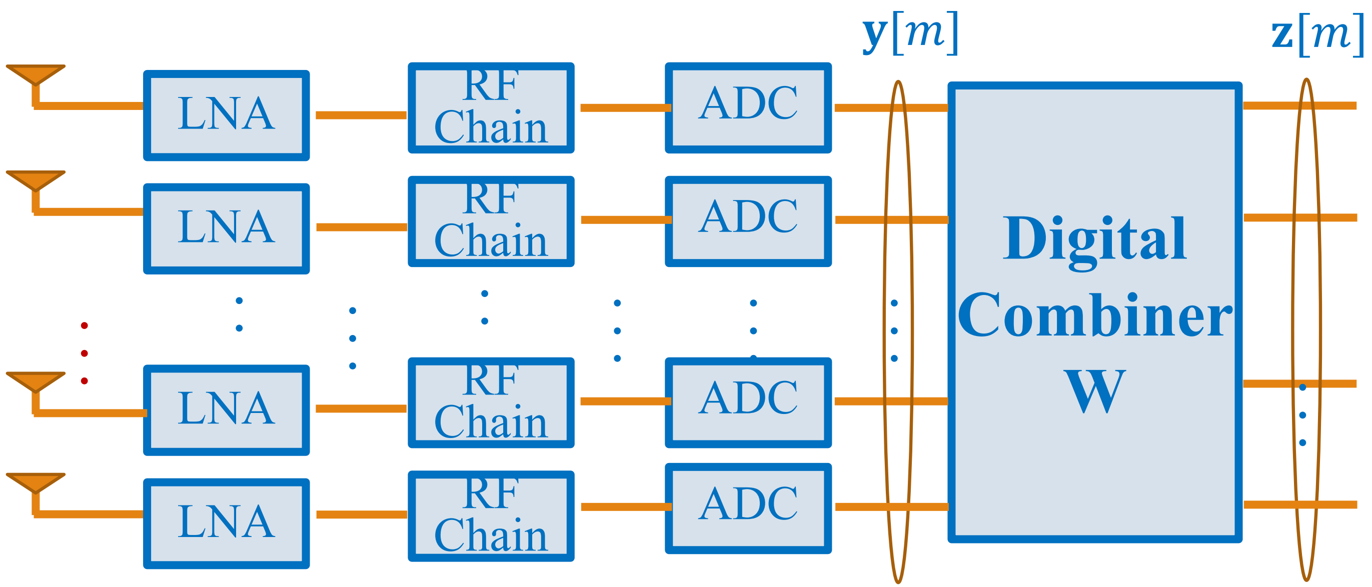
DAC : Digital to Analog Converter
DAC convers digital signal to an analog waveform

RF Chain:
RF= Radio Frequency
RF Chain up-convert baseband signal to a passband signal with a desired carrier frequency

LNA : Low Noise Amplifier
LNA amplifies the signal

Given SVD of channel matrix $\mathbf{H} = \mathbf{U\Sigma V}^H$,
the optimal precoding matrix is $\mathbf{F}_{\text{opt}} = \mathbf{V}$

Fully Digital Combiner at MIMO Receiver



LNA : Low Noise Amplifier

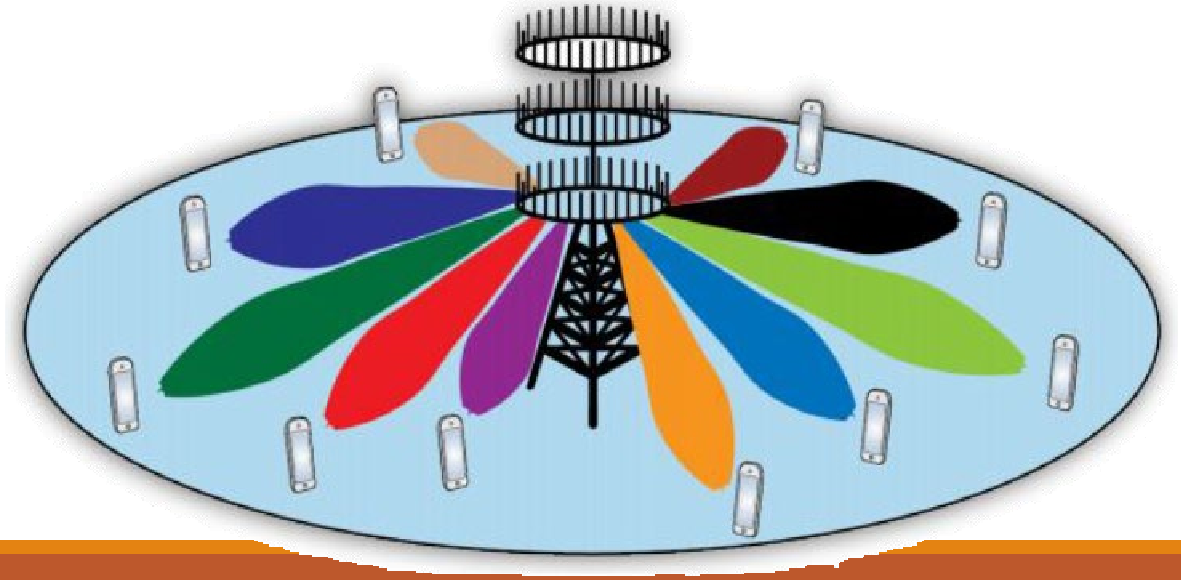
RF Chain:
RF= Radio Frequency
RF Chain down-convert passband signal to a baseband waveform

ADC : Analog to Digital Converter
ADC convers analog signal to a digital stream

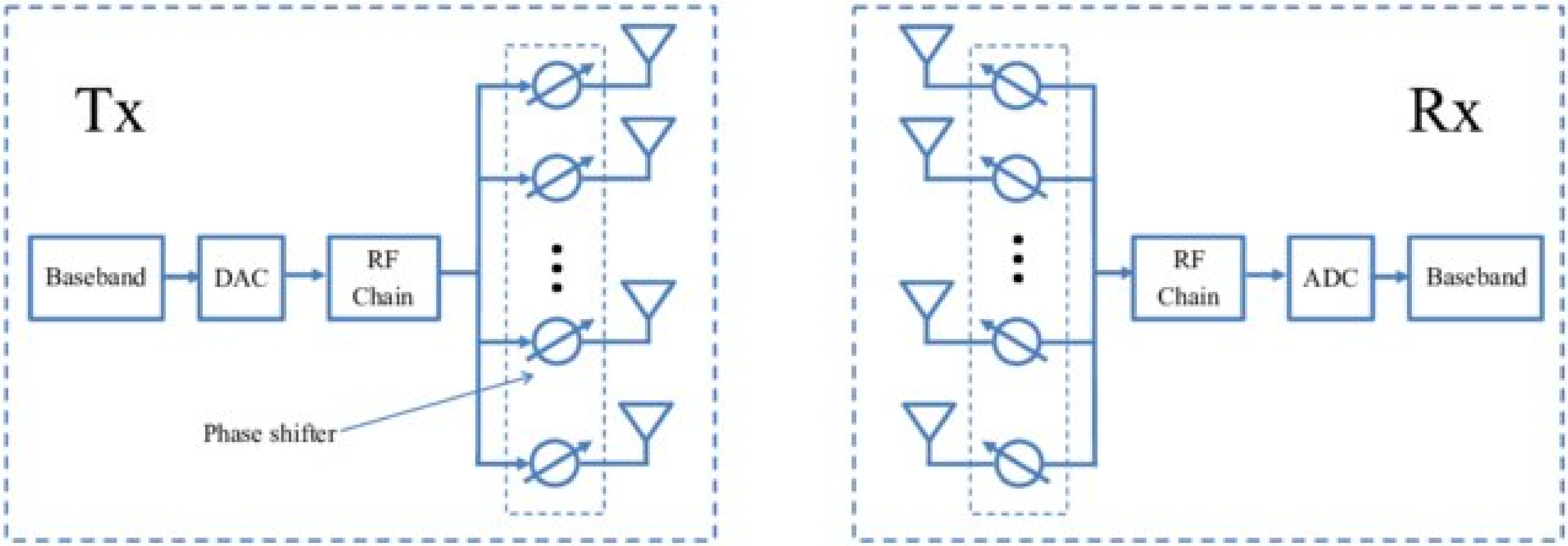
Given SVD of channel matrix $\mathbf{H} = \mathbf{U\Sigma V}^H$,
the optimal combining matrix is $\mathbf{W}_{\text{opt}} = \mathbf{U}^H$

Challenges of Precoding in Massive MIMO

- In Massive MIMO systems with fully digital precoder / combiner
- Number of RF chains, ADCs/DACs and LNAs is identical to Number of antennas
  1. The cost of RF chains and ADCs/DACs are higher especiallyfor mmWave devices
  2. It demands more volume to allocate numerous circuits of RF chain and ADC/DAC
- To tackle the challenges, we may consider
  - Analog beamforming
  - Hybrid Precoder/ Combiner



Analog Beamforming at Tx and Rx

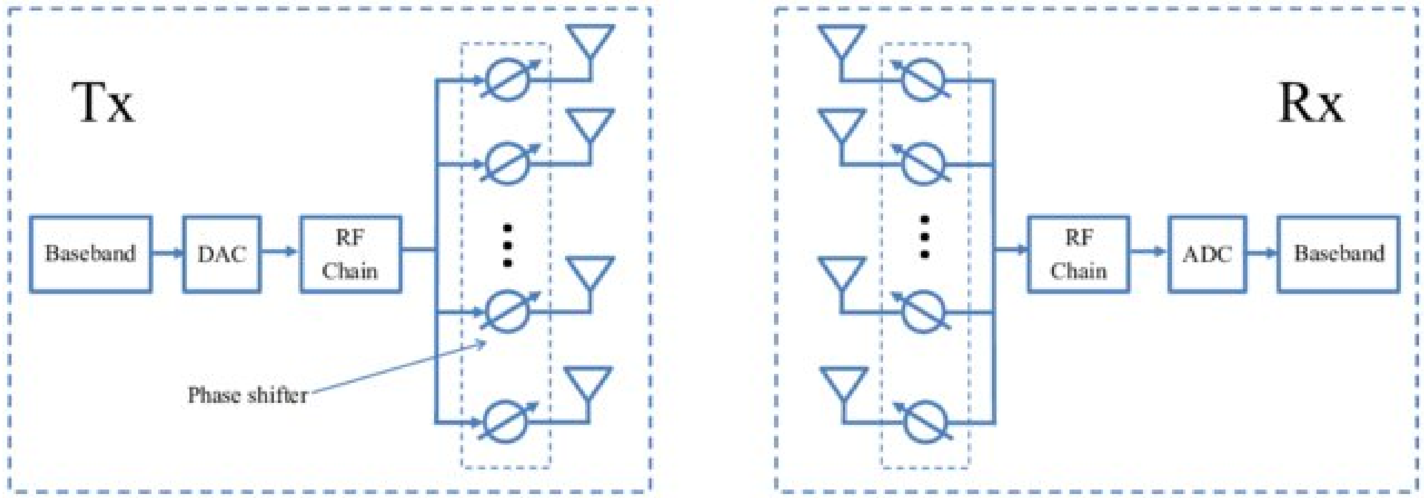


Analog Beamforming at Tx and Rx

Baseband received signal

$$y[m] = \mathbf{w}^H \mathbf{H} \mathbf{f} x[m] + n[m]$$

- ✓ Analog beamforming vector $\mathbf{f} = \frac{1}{\sqrt{N_t}} \begin{bmatrix} e^{j\varphi_1} & e^{j\varphi_2} & \dots & e^{j\varphi_{N_t}} \end{bmatrix}^T$
- ✓ Analog combining vector $\mathbf{w} = \frac{1}{\sqrt{N_r}} \begin{bmatrix} e^{j\phi_1} & e^{j\phi_2} & \dots & e^{j\phi_{N_r}} \end{bmatrix}^T$



Analog Beamforming at Tx and Rx

The vector $\mathbf{f}$ and $\mathbf{w}$ is designed to maximize SNR

$$\left(\mathbf{f}^{(\mathrm{opt})}, \mathbf{w}^{(\mathrm{opt})}\right) = \mathrm{argmax} \left|\mathbf{w}^{H}\mathbf{H}\mathbf{f}\right|^{2}$$

$$\text{s.t.} \quad |f_i| = 1/\sqrt{N_t}\,, i = 1,2, \dots, N_t$$

$$|w_i| = 1/\sqrt{N_r}\,, i = 1,2, \dots, N_r$$

- ✓ If there is no modulus constraint, the optimal solution is

$$\mathbf{f}^{(\mathrm{opt})} = \mathbf{V}(:,1)$$

$$\mathbf{w}^{(\mathrm{opt})} = \mathbf{U}(:,1)$$

- ✓ Matrices $\mathbf{U}$ and $\mathbf{V}$ are singular matrices of the channel matrix $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{H}$

Analog Beamforming at Tx and Rx

Under mmWave channel model

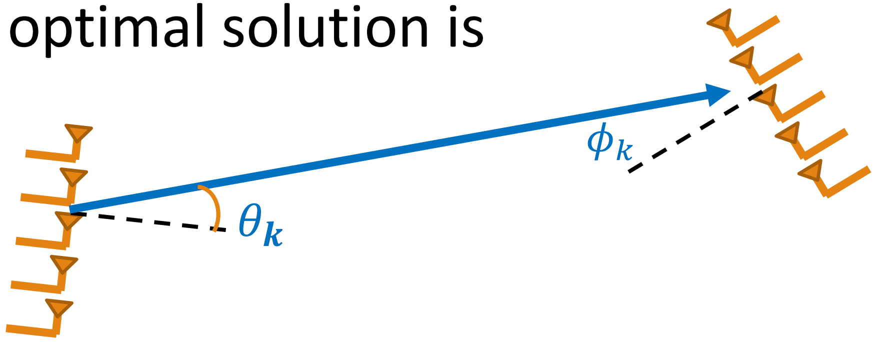
$$\mathbf{H} = \sqrt{PL^{-1} \cdot \psi} \sum_{\ell=0}^{L} \alpha_\ell \boldsymbol{a_r}(\phi_\ell) \boldsymbol{a_t}^H(\theta_\ell)$$

- ✓ $\boldsymbol{a_t}(\theta) = \frac{1}{\sqrt{N_t}} \left[1 \quad \exp(j\frac{2\pi}{\lambda} d_t \sin\theta) \quad \exp(j\frac{2\pi}{\lambda} 2d_t \sin\theta) \quad \cdots \quad \exp(j\frac{2\pi}{\lambda}(N_t - 1) d_t \sin\theta) \right]^T$
- ✓ $\boldsymbol{a_r}(\phi) = \frac{1}{\sqrt{N_r}} \left[1 \quad \exp(j\frac{2\pi}{\lambda} d_r \sin\phi) \quad \exp(j\frac{2\pi}{\lambda} 2d_r \sin\phi) \quad \cdots \quad \exp(j\frac{2\pi}{\lambda}(N_r - 1) d_r \sin\phi) \right]^T$
- ✓ If N_t and N_r are very large, it is proven that the optimal solution is

$$\mathbf{f}^{(\mathrm{opt})} = \boldsymbol{a_t}(\theta_k)$$

$$\mathbf{w}^{(\mathrm{opt})} = \boldsymbol{a_r}(\phi_k)$$

where k is selected from $k = \arg\max_\ell |\alpha_\ell|$



Remarks

- MIMO systems exploit spatial diversity and multiplexing gain effectively
- With perfect CSI, fully digital precoder and combiner are obtained from the SVD of the channel matrix
- If tx and rx have a single RF, analog precoder/ combiner can exploit array gain to enhance received SNR